

The **xint** bundle: **xint** and **xintgcd**

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Abstract

The **xint** package implements with expandable T EX macros the basic arithmetic operations of addition, subtraction, multiplication and division, as applied to arbitrarily long numbers represented as chains of digits with an optional minus sign.

The **xintgcd** package provides implementations of the Euclidean algorithm and of its typesetting.

The packages may be used with Plain and with L AT EX.

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1 Origins of this package

The package `bigintcalc` by HEIKO OBERDIEK already provides expandable arithmetic operations on “big numbers”, exceeding the \TeX limits (of $2^{31}-1$), so why another one?

I got started on this in early March 2013, via a thread on the `c.t.tex` usenet group, where ULRICH DIEZ used the previously cited package together with a macro (`\ReverseOrder`) which I had contributed to another thread.¹ What I had learned in this other thread thanks to interaction with ULRICH DIEZ and GL on expandable manipulations of tokens motivated me to try my hands at addition and multiplication.

¹The `\ReverseOrder` could be avoided in that circumstance, but it does play a crucial rôle here.

I wrote macros `\bigMul` and `\bigAdd` which I posted to the newsgroup; they appeared to work comparatively fast. These first versions did not use the ε - \TeX `\numexpr` macro, they worked one digit at a time, having previously stored digit arithmetic in (many) macros.

I noticed that the `bigintcalc` package used the `\numexpr` ε - \TeX primitive when available, but (as far as I could tell) not to do computations many digits at a time. Using `\numexpr` for one digit at a time for `\bigAdd` and `\bigMul` slowed them a tiny bit but avoided cluttering \TeX memory with 1200 macros storing pre-computed arithmetic with 2 or 3 digits. I wondered if some speed could be gained by using `\numexpr` to do four digits at a time for elementary multiplications (as the maximal admissible number for `\numexpr` has ten digits).

The present package is the result of this initial questioning.

xint requires the ε - \TeX `\numexpr` primitive.

I have aimed at speed wherever I could, and to the extent that I could guess what was more efficient for \TeX . After a while though I did opt for more readable coding style in those parts of the code which were not at the heart of repeatedly used loops. In particular I started using `\ifnum` and `\ifcase` constructs which I had completely avoided so far, working only with macro expansions.

I wrote a version of addition which does `\numexpr` operations eight digits at a time, but its additional overhead made it a bit slower for numbers of up to a few hundreds digits and it became faster only for numbers with thousands of digits; for such sizes multiplication starts taking a noticeable time, so I have chosen to retain the addition routine which was most efficient for numbers having a few dozens to a few hundreds digits.

This implementation is thus a \TeX nical thing, quite different from what one would do in a structured programming language like C, although the underlying algorithms are just the standard steps applied to hand computations (nothing fancy like Fast Fourier Transform...).

By the way, yes **xint** enjoys working fast and efficiently with 200 digits numbers, but surely any program (even poorly written) in C using the CPU for arithmetic operations on arrays of numbers (not digits!!!) will work thousands of times faster (or more, I don't know) than what can be achieved using \TeX to manipulate strings of ASCII representations of digits!

2 Expansions

Except otherwise stated all macros are completely expandable. For example, with the following code snippet within `myfile.tex`

```
\newwrite\outfile
\immediate\openout\outfile \jobname-out\relax
\immediate\write\outfile {\xintQuo{\xintPow{2}{1000}}{\xintFac{100}}}
% \immediate\closeout\outfile
```

the tex run creates a file `myfile-out.tex` containing the decimal representation of the integer quotient $2^{1000}/100!$. Such macros can also be used inside a `\csname ... \endcsname`, and of course in an `\edef`.

2 Expansions

Furthermore the package macros give their final results in two expansion steps. They twice expand their arguments so that they can be arbitrarily chained. Hence

```
\xintLen{\xintQuo{\xintPow{2}{1000}}{\xintFac{100}}}
```

expands in two steps and tells us that $[2^{1000}/100!]$ has 144 digits. This is not so many, let us print them here: 11481324964150750548227839387255106625980551778418617 288366347806582654189470473797041953579887663048435826506006150374953 1707793118627774829601. For the sake of typesetting this documentation and not have big numbers extend into the margin and go beyond the page physical limits, I use this little macro (not provided by the package):

```
\def\allownumbersplit #1 {\ifx #1\relax \else #1\hspace{0pt plus 1pt} \\expandafter\allownumbersplit\fi}%
```

To provoke the double expansion first, it is used as in:

```
\expandafter\expandafter\expandafter\allownumbersplit \\xintQuo{\xintPow{2}{1000}}{\xintFac{100}}\relax
```

Or, the computation can be done inside an `\edef` and then only one `\expandafter` will be enough before `\allownumbersplit`.

Remarks on the double expansion of arguments:

- When I say that the macros expand twice their arguments, this means that they expand the first token seen (for each argument), then expand again the first token of the result of the first expansion. For example

```
\def\x{12}\def\y{34}\xintAdd {\x}{\x\y}
```

is *not* a legal construct. It works here by sheer luck as the `\y` gets expanded inside a `\numexpr`. But this would fail in general: if you need a more complete (expandable...) expansion of your initial input, you should use the `\bigintcalcNum` macro from the `bigintcalc` package. Or, outside of an expandable-only context, just massage your inputs through `\edef`'s.

- Unfortunately, after `\def\x {12}`, one can not use just `-\x` as input to one of the package macros: the rules above explain that the twice expansion will act only on the minus sign, hence do nothing. The only way is to use the `\xintOpp` macro, which replaces a number with its opposite. Example: `\xintAdd {\xintOpp\x}{\x}=0`.

- With the definition

```
\def\AplusBC #1#2#3{\xintAdd {#1}{\xintMul {#2}{#3}}}
```

one obtains an expandable macro producing the expected result, not in two, but rather in three steps: a first expansion is consumed by the macro expanding to its definition. As a result `\xintAdd {\AplusBC {1}{2}{3}}{4}` would then miserably fail. The solution is to use the *lowercase* form of `\xintAdd`:

```
\def\AplusBC #1#2#3{\romannumeral0\xintadd {#1}{\xintMul {#2}{#3}}}
```

and then `\AplusBC` will share the same properties as do the other `xint` ‘primitive’ macros.

Don’t leave any space after the zero, and use the lowercase form *only* for the external highest level of chained commands. All `xint` provided public macros have such a lowercase form precisely to facilitate building-up higher level macros based on them.

3 Inputs

After a twice expansion of the arguments, the ensuing numbers have to be strings of digits with one (and not more) optional minus sign (and not a plus sign). The first digit is not zero if there are more than one digit. And `-0` is not legal input. Syntax such as `\xintMul\A\B` is accepted and equivalent to `\xintMul {\A}{\B}`. Of course `\xintAdd\xintMul\A\B\C` does not work, the product operation must be put within braces: `\xintAdd{\xintMul\A\B}\C`.

It would be nice to have a functional form `\add(x, \mul(y, z))` but this is not provided by the package. Arguments must be either within braces or a single control sequence.

For the division (but not for addition, subtraction, or multiplication), the two inputs must have at most $2^{31}-9=2147483639$ digits.

I guess anyhow that this is way way way beyond what is possible in terms of memory in any implementation of \TeX . But if the situation did arise nevertheless of such a gigantic input, an arithmetic overflow would occur (after some long time I guess) as `xint` first computes the lengths of the inputs by using `\numexpr` with successive additions of the number 8 to itself until the whole input has been parsed² (this initial step is only for the division algorithm, the three other arithmetic operations remain unaware of the sizes of their inputs, although they do experience them in a sense, as they initially reverse the order of digits of at least one of the inputs, which means they have to scan it entirely).

Also: the factorial function `\xintFac` will refuse to (start...) compute $N!$ if $N \geq 1000000000$, and the power function `\xintPow {A}{B}`, when the absolute value $|A|$ is at least two, will refuse to start the computation if $B \geq 1000000000$ (the minimal outcome is $2^{1000000000}$ which has 301029996 digits...).

In those latter cases, no arithmetic overflow will happen, but rather, copied from package `bigintcalc`, undefined control sequences with names indicating the source of the problem are inserted in the token stream and will appear in the log file in \TeX ‘undefined macro’ error messages. This will not stop the computation, which (most of the time) will output a zero.

No check is done on the format of the inputs after the initial twice expansion. Often, but not always, something starting with a `0` will be assumed to be zero (throwing the rest away, or sometimes not which then will lead to errors). Plus signs are not accepted and will cause errors.

The sole exception is the macro `\xintNum` which accepts numbers starting with an arbitrary long sequence of plus signs, minus signs, followed by zeros and will remove all of them, keeping only the correct sign:

```
\xintNum {-----0000000009876543210}=-9876543210
```

But don’t insert zeros within the initial signs. An empty string is also acceptable input: `\xintNum {}=0`. As with all other package macros, `\xintNum` expands twice its argument, and obtains its final result in two expansion steps.

²It is the macro `\xintLen` (used by `\xintDivision`) which will trigger an arithmetic overflow if it is called with an input of more than 2147483639 digits. I thought it was not worthwhile adding to the code of `\xintLen` a safeguard against the arithmetic overflow in a `\numexpr`: this check would have some general impact on speed, whereas the situation can not realistically occur (or even not at all, I admit not having double-checked the intrinsic \TeX memory limitations).

\TeX 's count registers cannot be directly used but must be prefixed by \the or \number . The same for \numexpr expressions.

4 Outputs

The output, when it consists of a single number, is always in the normalized form described in the previous section. Some macros have an output consisting of more than one number, each one is then within braces. For example `\xintDivision` gives first the quotient and then the remainder, each of them within braces. This is for programming purposes to avoid having to do twice the division, once for the quotient, the other one for the remainder: but of course macros `\xintQuo` and `\xintRem` are provided for easier direct access.

The macro `\xintDecSplit{x}{N}`³ cuts its second argument N at a location specified by its first argument x , and returns the two pieces one after the other, each within braces. Depending on the value of x and the length of N , the first, or the second, output of `\xintDecSplit` may be *empty*. Leading zeros in the second string of digits are neither removed. This is the only situation where a package macro may output something which would need to be input to `\xintNum` before further processing by the other package macros.

When using things such as `\ifcase \xintSgn{\A}` one has to leave a space after the closing brace for \TeX to stop its scanning for a number: once \TeX has finished expanding `\xintSgn{\A}` and has so far obtained either 1, 0, or -1, a space (or something ‘unexpandable’) must stop it looking for more digits. Using `\ifcase\xintSgn\A` without the braces is very dangerous, because the blanks (including the end of line) following `\A` will be skipped and not serve to stop the number which `\ifcase` is looking for. With `\def\A{1}`:

```
\ifcase \xintSgn\A 0\or OK\else ERROR\fi    ---> gives ERROR
\ifcase \xintSgn{\A} 0\or OK\else ERROR\fi   ---> gives OK
```

5 Assignments

You might not need to maintain at all times complete expandability. For example why not allow oneself the two definitions `\edef\A {\xintQuo{100}{3}}` and `\edef\B {\xintRem {100}{3}}`. A special syntax is provided to make these things more efficient, as the package provides `\xintDivision` which computes both quotient and remainder at the same time:

```
\xintAssign\xintDivision{100}{3}\to\A\B
\xintAssign\xintDivision{\xintPow {2}{1000}}{\xintFac{100}}\to\A\B
gives \meaning\A: macro:->1148132496415075054822783938725510662598055177
841861728836634780658265418947047379704195357988766304843582650600061503
749531707793118627774829601 and \meaning\B: macro:->54936294521339832251
38128786223912807341050049847605059532189961231327664902288388132878702
444582075129603152041054804964625083138567652624386837205668069376.
```

Another example (which uses a macro from the `xintgcd` package):

```
\xintAssign\xintBezout{357}{323}\to\A\B\U\V\D
```

³Its behavior has been modified in bundle version 1.01, check its documentation.

5 Assignments

is equivalent to setting \mathbf{A} to 357, \mathbf{B} to 323, \mathbf{U} to -9, \mathbf{V} to -10, and \mathbf{D} to 17. And indeed $(-9) \times 357 - (-10) \times 323 = 17$ is a Bezout Identity.

```
\xintAssign\xintBezout{357}{902836026}{200467139463}\to\A\B\U\V\mathbf{D}
gives then \mathbf{U}: macro:->5812117166, \mathbf{V}: macro:->103530711951 and \mathbf{D}=3.
```

When one does not know in advance the number of tokens, one can use `\xintAssignArray` or its synonym `\xintDigitsOf`:

```
\xintDigitsOf\xintPow{2}{100}\to\Out
```

This defines `\Out` to be macro with one parameter, `\Out{0}` gives the size N of the array and `\Out{n}`, for n from 1 to N then gives the n th element of the array, here the n th digit of 2^{100} , from the most significant to the least significant. As usual, the generated macro `\Out` is completely expandable and expands twice its (unique) argument. Consider the following code snippet:

```
\newcount\cnta
\newcount\cntb
\begin{group}
\xintDigitsOf\xintPow{2}{100}\to\Out
\cnta = 1
\cntb = 0
\loop
\advance\cntb \xintSqr{\Out{\the\cnta}}
\ifnum\cnta < \Out{0}
\advance\cnta 1
\repeat

| 2^{100} | (= \xintPow{2}{100}) has \Out{0} digits and the sum of
their squares is \the\cntb. These digits are, from the least to
the most significant: \cnta = \Out{0}
\loop \Out{\the\cnta} \ifnum\cnta > 1 \advance\cnta -1, \repeat.
\end{group}
```

2^{100} ($= 1267650600228229401496703205376$) has 31 digits and the sum of their squares is 679. These digits are, from the least to the most significant: 6, 7, 3, 5, 0, 2, 3, 0, 7, 6, 9, 4, 1, 0, 4, 9, 2, 2, 8, 2, 2, 0, 0, 6, 0, 5, 6, 7, 6, 2, 1.

We used a group in order to release the memory taken by the `\Out` array: indeed internally, besides `\Out` itself, additional macros are defined which are `\Out0`, `\Out00`, `\Out1`, `\Out2`, ..., `\OutN`, where N is the size of the array (which is the value returned by `\Out{0}`; the digits are parts of the names not arguments).

The command `\xintRelaxArray\Out` sets all these macros to `\relax`, but it was simpler to put everything withing a group.

Needless to say `\xintAssign`, `\xintAssignArray` and `\xintDigitsOf` do not do any check on whether the macros they define are already defined.

In the example above, we deliberately broke all rules of complete expandability, but had we wanted to compute the sum of the digits, not the sum of the squares, we could just have written:

```
\xintSum{\xintPow{2}{100}}=115
```

Indeed, `\xintSum` is usually used as in

```
\xintSum{{123}{-345}{\xintFac{7}}{\xintOpp{\xintRem{3347}{591}}}}=4426
but in the example above each digit of  $2^{100}$  is treated as would have been a summand
enclosed within braces, due to the rules of TeX for parsing macro arguments.
```

6 Error messages

Note that `{-\xintRem{3347}{591}}` is not a valid input, because the double expansion will apply only to the minus sign and leave unaffected the `\xintRem`. So we used `\xintOpp` which replaces a number with its opposite.

As a last example of use of `\xintAssignArray` here is one line from the source code of the `xintgcd` macro `\xintTypesetEuclideAlgorithm`:

```
\xintAssignArray\xintEuclideAlgorithm {\#1}{\#2}\to\U
```

This is done inside a group. After this command `\U{1}` contains the number N of steps of the algorithm (not to be confused with $\U{0}=2N+4$ which is the number of elements in the `\U` array), and the GCD is to be found in `\U{3}`, a convenient location between `\U{2}` and `\U{4}` which are (absolute values of the twice expansion of) the initial inputs. Then follow N quotients and remainders from the first to the last step of the algorithm. The `\xintTypesetEuclideAlgorithm` macro organizes this data for typesetting: this is just an example of one way to do it.

6 Error messages

We employ the same method as in the `bignumcalc` package. But the error is always thrown *before* the end of the `\romannumeral0` expansion so as to not disturb further processing of the token stream, if the operation was a secondary one whose output is expected by a first one. Here is the list of possible errors:

```
\xintError:ArrayIndexIsNegative  
\xintError:ArrayIndexBeyondLimit  
\xintError:FactorialOfNegativeNumber  
\xintError:FactorialOfTooBigNumber  
\xintError:DivisionByZero  
\xintError:FractionRoundedToZero  
\xintError:ExponentTooBig  
\xintError:TooBigDecimalShift  
\xintError:TooBigDecimalSplit  
\xintError>NoBezoutForZeros
```

7 Package namespace

Inner macros of the `xint` and `xintgcd` packages all begin either with `\XINT@` or with `\xint@`. The package public commands all start with `\xint`. The major forms have their initials capitalized, and lowercase forms, prefixed with `\romannumeral0`, allow definitions of further macros expanding in two steps to their full expansion (and can thus be chained with the ‘primitive’ `xint` macros). Some other control sequence names are used only as delimiters, and left undefined.

The `\xintReverseOrder{\langle tokens \rangle}` macro uses `\xint@UNDEF` and `\xint@undef` as dummy tokens and can be used on arbitrary token strings not containing these control sequence names. Anything within braces is treated as one unit: one level of exterior braces is removed and the contents are not reverted.

8 Loading and usage

Usage with LaTeX: `\usepackage{xint}`
`\usepackage{xintgcd}`

Usage with TeX: `\input xint.sty\relax`
`\input xintgcd.sty\relax`

We have added, directly copied from packages by HEIKO OBERDIEK, a mechanism of reload and ε -TeX detection, especially for Plain TeX. As ε -TeX is required, the executable `tex` can not be used, `etex` or `pdftex` (version 1.40 or later) or ..., must be invoked.

Furthermore, the package `xintgcd` will check for previous loading of `xint`, and will try to load it if this was not already done.

Also inspired from the HEIKO OBERDIEK packages we have included a complete catcode protection mechanism. The packages may be loaded in any catcode configuration satisfying these requirements: the percent is comment character, the backslash is escape character, digits have category code other and letters have category code letter. Nothing else is assumed, and the previous configuration is restored after the loading of the packages.

This is for the loading of the packages. For the actual use of the macros, note that when feeding them with negative numbers the minus sign must have category code other, as is standard.

`xint` presupposes that the usual `\space` and `\empty` macros are pre-defined, which is the case in Plain TeX as well as in L^AT_EX.

Lastly, the macros `\xintRelaxArray` (of `xint`) and `\xintTypesetEuclideAlgorithm` and `\xintTypesetBezoutAlgorithm` (of `xintgcd`) use `\loop`, both Plain and L^AT_EX incarnations are compatible. `\xintTypesetBezoutAlgorithm` also uses the `\endgraf` macro.

9 Installation

Run `tex` or `latex` on `xint.dtx`.

This will extract the style files `xint.sty` and `xintgcd.sty` (and `xint.ins`). Files with the same names and in the same repertory will be overwritten. The `tex` (not `latex`) run will stop with the complaint that it does not understand `\NeedsTeXFormat`, but the style files will already have been extracted by that time.

Alternatively, run `tex` or `latex` on `xint.ins` if available.

To get `xint.pdf` run `pdflatex` thrice on `xint.dtx`

```
xint.sty, xintgcd.sty -> TDS:tex/generic/xint/
xint.dtx                  -> TDS:source/generic/xint/
xint.pdf                  -> TDS:doc/generic/xint/
```

It may well be necessary to then refresh the TeX installation filename database.

10 Commands of the **xint** package

{N} (resp. {M} or {x}) stands for a normalised number within braces as described in the documentation, or for a control sequence expanding in at most two steps to such a number (without the braces!), or for a control sequence within braces expanding in at most two steps to such a number, or for material within braces which expands in two expansion of the first token to such a number.

10.1 **\xintRev**

`\xintRev{N}` will revert the order of the digits of the number, keeping the optional sign. Leading zeros resulting from the operation are not removed (see the `\xintNum` macro for this).

```
\xintRev{-123000}=-000321
\xintNum{\xintRev{-123000}}=-321
```

10.2 **\xintReverseOrder**

`\xintReverseOrder{<token_list>}` does not do any expansion of its argument and just reverses the order of the tokens. Brace pairs encountered are removed once and the enclosed material does not get reverted.

```
\xintReverseOrder{\xintDigitsOf\xintPow {2}{100}\to\Stuff}
gives: \Stuff \to 1002\xintPow \xintDigitsOf
```

10.3 **\xintNum**

`\xintNum{N}` removes chains of plus or minus signs, followed by zeros.

```
\xintNum{-----00000000367941789479}=-367941789479
```

10.4 **\xintLen**

`\xintLen{N}` returns the length of the number, not counting the sign.

```
\xintLen{-12345678901234567890123456789}=29
```

10.5 **\xintLength**

`\xintLength{<token_list>}` does not do any expansion of its argument and just counts how many tokens there are. Things enclosed in braces count as one.

```
\xintLength {\xintPow {2}{100}}=3
≠ \xintLen {\xintPow {2}{100}}=31
```

10.6 **\xintAssign**

`\xintAssign<braced things>\to<as many cs as they are things>` defines (without checking if something gets overwritten) the control sequences on the right of `\to` to be the complete expansions of the successive things on the left of `\to` enclosed within braces.

Important: a double expansion is applied first to the material extending up to `\to`.

As a special exception, if after this initial double expansion a brace does not immediately follows `\xintAssign`, it is assumed that there is only one control sequence to define and it is then defined to be the complete expansion of the material between `\xintAssign` and `\to`.

```
\xintAssign\xintDivision{100000000000}{133333333}\to\Q\R
    \meaning\Q: macro:->7500, \meaning\R: macro:->2500
    \xintAssign\xintPow {7}{13}\to\SevenToThePowerThirteen
        \SevenToThePowerThirteen=96889010407
```

Of course this macro and its cousins completely break usage in pure expansion contexts, as assignments are made via the `\edef` primitive.

10.7 `\xintAssignArray`

`\xintAssignArray<braced things>\to\myArray` first double expands the first token then defines `\myArray` to be a macro with one parameter, such that `\myArray{N}` expands in two steps (which include the twice-expansion of `{N}`) to give the Nth braced thing, itself completely expanded. `\myArray{0}` returns the number M of elements of the array so that the successive elements are `\myArray{1}, ..., \myArray{M}`.

```
\xintAssignArray\xintBezout {1000}{113}\to\Bez
```

will set `\Bez{0}` to 5, `\Bez{1}` to 1000, `\Bez{2}` to 113, `\Bez{3}` to -20, `\Bez{4}` to -177, and `\Bez{5}` to 1: $(-20) \times 1000 - (-177) \times 113 = 1$.

10.8 `\xintRelaxArray`

`\xintRelaxArray\myArray` sets to `\relax` all macros which were defined by the previous `\xintAssignArray` with `\myArray` as array name.

10.9 `\xintDigitsOf`

This is a synonym for `\xintAssignArray`, to be used to define an array giving all the digits of a given number.

```
\xintDigitsOf\xintPow {7}{500}\to\digits
```

7^{500} has `\digits{0}=423` digits, and the 123rd among them (starting from the most significant) is `\digits{123}=3`.

10.10 `\xintSgn`

`\xintSgn{N}` returns 1 if the number is positive, 0 if it is zero and -1 if it is negative.

10.11 `\xintOpp`

`\xintOpp{N}` returns the opposite $-N$ of the number N .

10.12 `\xintAbs`

`\xintAbs{N}` returns the absolute value of the number.

10.13 \xintAdd

`\xintAdd{N}{M}` returns the sum of the two numbers. It is more efficient to have the longer of the two be the first argument.

10.14 \xintSub

`\xintSub{N}{M}` returns the difference $N-M$.

10.15 \xintCmp

`\xintCmp{N}{M}` returns 1 if $N > M$, 0 if $N = M$, and -1 if $N < M$.

10.16 \xintGeq

`\xintGeq{N}{M}` returns 1 if the absolute value of the first number is at least equal to the absolute value of the second number. If $|N| < |M|$ it returns 0.

10.17 \xintMax

`\xintMax{N}{M}` returns the largest of the two in the sense of the order structure on the relative integers (*i.e.* the right-most number if they are put on a line with positive numbers on the right).

10.18 \xintMin

`\xintMin{N}{M}` returns the smallest of the two in the sense of the order structure on the relative integers (*i.e.* the left-most number if they are put on a line with positive numbers on the right).

10.19 \xintSum

`\xintSum{<braced things>}` after expanding its argument twice expects to find a sequence of tokens (or braced material). Each is twice-expanded, and the sum of all these numbers is returned.

```
\xintSum{{123}{-98763450}{\xintFac{7}}{\xintMul{3347}{591}}}= -96780210
                                         \xintSum{1234567890}=45
```

An empty sum is no error and returns zero: `\xintSum {}=0`. A sum with only one term returns that number: `\xintSum {{-1234}}=-1234`. Attention that `\xintSum {-1234}` is not legal input and will make the TeX run fail. On the other hand `\xintSum {1234}=10`.

10.20 \xintSumExpr

`\xintSum{braced things}\relax` is to what `\xintSum` reduces after its initial double expansion of its argument.

```
\xintSumExpr {123}{-98763450}{\xintFac{7}}{\xintMul{3347}{591}}\relax= -96780210
```

10.21 \xintMul

`\xintMul{N}{M}` returns the product of the two numbers. It is more efficient to have the first one be the longer of the two.

10.22 \xintSqr

`\xintSqr{N}` returns the square.

10.23 \xintPrd

`\xintPrd{\langle braced things \rangle}` after expanding its argument twice expects to find a sequence of tokens (or braced material). Each is twice-expanded, and the product of all these numbers is returned.

```
\xintPrd{{-9876}}{\xintFac{7}}{\xintMul{3347}{591}}=-98458861798080
\xintPrd{123456789123456789}=131681894400
```

An empty product is no error and returns 1: `\xintPrd {}=1`. A product reduced to a single term returns this number: `\xintPrd {{-1234}}=-1234`. Attention that `\xintPrd {-1234}` is not legal input and will make the TeX compilation fail. On the other hand `\xintPrd {1234}=24`.

10.24 \xintProductExpr

`\xintProductExpr{\langle braced things \rangle}\relax` is to what `\xintPrd` reduces after its initial double expansion of its argument.

```
\xintProductExpr 123456789123456789\relax=131681894400
```

10.25 \xintFac

`\xintFac{N}` returns the factorial. It is an error if the argument is negative or at least 10^9 . It is not recommended to launch the computation of things such as $100000!$, if you need your computer for other tasks.

10.26 \xintPow

`\xintPow{N}{M}` returns N^M . When M is zero, this is 1. Some cases (N zero and M negative, $|N|>1$ and M negative, $|N|>1$ and M at least 10^9) make **xint** throw errors.

10.27 \xintDivision

`\xintDivision{N}{M}` returns `{quotient Q}{remainder R}`. This is euclidean division: $N = QM + R$, $0 \leq R < |M|$. So the remainder is always non-negative and the formula $N = QM + R$ always holds independently of the signs of N or M . Division by zero is of course an error (even if N vanishes) and returns `{0}{0}`.

10.28 \xintQuo

`\xintQuo{N}{M}` returns the quotient from the euclidean division.

10.29 \xintRem

`\xintRem{N}{M}` returns the remainder from the euclidean division.

10.30 \xintFDg

`\xintFDg{N}` returns the first digit (most significant) of the decimal expansion.

10.31 \xintLDg

`\xintLDg{N}` returns the least significant digit. When the number is positive, this is the same as the remainder in the euclidean division by ten.

10.32 \xintOdd

`\xintOdd{N}` is 1 if the number is odd and 0 otherwise.

10.33 \xintDSL

`\xintDSL{N}` is decimal shift left, *i.e.* multiplication by ten.

10.34 \xintDSR

`\xintDSR{N}` is decimal shift right, *i.e.* it removes the last digit (keeping the sign). For a positive number, this is the same as the quotient from the euclidean division by ten (of course, done in a more efficient manner than via the general division algorithm). For N from -9 to -1, the macro returns 0.

10.35 \xintDSH

`\xintDSH{x}{N}` is parametrized decimal shift. When x is negative, it is like iterating `\xintDSL{|x|}` times (*i.e.* multiplication by $10^{-|x|}$). When x positive, it is like iterating `\xintDSR{x}` times (and is more efficient of course), and for a non-negative N this is thus the same as the quotient from the euclidean division by 10^x .

10.36 \xintDSHr, \xintDSx

New in bundle version 1.01.

`\xintDSHr{x}{N}` expects x to be zero or positive and it returns then a value R which is correlated to the value Q returned by `\xintDSH{x}{N}` in the following manner:

- if N is positive or zero, Q and R are the quotient and remainder in the euclidean division by 10^x (obtained in a more efficient manner than using `\xintDivision`),
- if N is negative let Q1 and R1 be the quotient and remainder in the euclidean division by 10^x of the absolute value of N. If Q1 does not vanish, then Q=-Q1 and R=R1. If Q1 vanishes, then Q=0 and R=-R1.
- for x=0, Q=N and R=0.

So one has $N = 10^x Q + R$ if Q turns out to be zero or positive, and $N = 10^x Q - R$ if Q turns out to be negative, which is exactly the case when N is at most -10^x .

$\text{\xintDSx}\{x\}\{N\}$ for x negative is exactly as $\text{\xintDSH}\{x\}\{N\}$, *i.e.* multiplication by 10^{-x} . For x zero or positive it returns the two numbers $\{Q\}\{R\}$ described above, each one within braces. So Q is $\text{\xintDSH}\{x\}\{N\}$, and R is $\text{\xintDSHr}\{x\}\{N\}$, but computed simultaneously.

```
\xintAssign\xintDSx {-1}{-123456789}\to\N
\meaning\N: macro:->-1234567890.
\xintAssign\xintDSx {-20}{123456789}\to\N
\meaning\N: macro:->12345676890000000000000000000000.
\xintAssign\xintDSx {0}{-123004321}\to\Q\R
\meaning\Q: macro:->-123004321, \meaning\R: macro:->0.
\xintDSH {0}{-123004321}=-123004321, \xintDSHr {0}{-123004321}=0
\xintAssign\xintDSx {6}{-123004321}\to\Q\R
\meaning\Q: macro:->-123, \meaning\R: macro:->4321.
\xintDSH {6}{-123004321}=-123, \xintDSHr {6}{-123004321}=4321
\xintAssign\xintDSx {8}{-123004321}\to\Q\R
\meaning\Q: macro:->-1, \meaning\R: macro:->23004321.
\xintDSH {8}{-123004321}=-1, \xintDSHr {8}{-123004321}=23004321
\xintAssign\xintDSx {9}{-123004321}\to\Q\R
\meaning\Q: macro:->0, \meaning\R: macro:->-123004321.
\xintDSH {9}{-123004321}=0, \xintDSHr {9}{-123004321}=-123004321
```

10.37 **\xintDecSplit**

Modified in bundle version 1.01!

$\text{\xintDecSplit}\{x\}\{N\}$ cuts the number into two pieces (each one within a pair of enclosing braces). First the sign if present is *removed*. Then, for x positive or null, the second piece contains the x least significant digits (*empty* if $x=0$) and the first piece the remaining digits (*empty* when x equals or exceeds the length of N). Leading zeros in the second piece are not removed. When x is negative the first piece contains the $|x|$ most significant digits and the second piece the remaining digits (*empty* if x equals or exceeds the length of N). Leading zeros in this second piece are not removed. So the absolute value of the original number is always the concatenation of the first and second piece.

This macro is for use in future components of the **xint** bundle. Its behavior for N non-negative is final and will not change. I am still hesitant about what to do with the sign of a negative N . It is recommended to use the macro only for non-negative N until the definitive version is released.

```
\xintAssign\xintDecSplit {0}{-123004321}\to\L\R
\meaning\L: macro:->123004321, \meaning\R: macro:->.
\xintAssign\xintDecSplit {5}{-123004321}\to\L\R
\meaning\L: macro:->1230, \meaning\R: macro:->04321.
\xintAssign\xintDecSplit {9}{-123004321}\to\L\R
\meaning\L: macro:->, \meaning\R: macro:->123004321.
\xintAssign\xintDecSplit {10}{-123004321}\to\L\R
\meaning\L: macro:->, \meaning\R: macro:->123004321.
\xintAssign\xintDecSplit {-5}{-12300004321}\to\L\R
```

11 Commands of the **xintgcd** package

```
\meaning\L: macro:->12300, \meaning\R: macro:->004321.  
          \xintAssign\xintDecSplit {-11}{-12300004321}\to\L\R  
\meaning\L: macro:->12300004321, \meaning\R: macro:->.  
          \xintAssign\xintDecSplit {-15}{-12300004321}\to\L\R  
\meaning\L: macro:->12300004321, \meaning\R: macro:->.
```

10.38 **\xintDecSplitL**

`\xintDecSplitL{x}{N}` returns the first piece after the action of `\xintDecSplit`.

10.39 **\xintDecSplitR**

`\xintDecSplitR{x}{N}` returns the second piece after the action of `\xintDecSplit`.

11 Commands of the **xintgcd** package

11.1 **\xintGCD**

`\xintGCD{N}{M}` computes the greatest common divisor. It is positive, except when both `N` and `M` vanish, in which case the macro returns zero.

```
\xintGCD{10000}{1113}=1  
\xintGCD{123456789012345}{9876543210321}=3
```

11.2 **\xintBezout**

`\xintBezout{N}{M}` returns five numbers `A`, `B`, `U`, `V`, `D` within braces. `A` is the first (twice-expanded) input number, `B` the second, `D` is the GCD, and $UA - VB = D$.

```
\xintAssign {{\xintBezout {10000}{1113}}}\to\X  
\meaning\X: macro:->{10000}{1113}{-131}{-1177}{1}.  
\xintAssign {\xintBezout {10000}{1113}}\to\A\B\U\V\D  
\A: 10000, \B: 1113, \U: -131, \V: -1177, \D: 1.  
\xintAssign {\xintBezout {123456789012345}{9876543210321}}\to\A\B\U\V\D  
\A: 123456789012345, \B: 9876543210321, \U: 256654313730, \V: 3208178892607,  
\D: 3.
```

11.3 **\xintEuclideAlgorithm**

`\xintEuclideAlgorithm{N}{M}` applies the Euclidean algorithm and keeps a copy of all quotients and remainders.

```
\xintAssign {{\xintEuclideAlgorithm {10000}{1113}}}\to\X  
\meaning\X: macro:->{5}{10000}{1}{1113}{8}{1096}{1}{17}{64}{8}{2}{1}{8}{0}. The first token is the number of steps, the second is N, the third is the GCD, the fourth is M then the first quotient and remainder, the second quotient and remainder, ... until the final quotient and last (zero) remainder.
```

11.4 \xintBezoutAlgorithm

`\xintBezoutAlgorithm{N}{M}` applies the Euclidean algorithm and keeps a copy of all quotients and remainders. Furthermore it computes the entries of the successive products of the 2 by 2 matrices $\begin{pmatrix} q & 1 \\ 1 & 0 \end{pmatrix}$ formed from the quotients arising in the algorithm.

```
\xintAssign {{\xintEuclideanAlgorithm {10000}{1113}}}\to\x
\meaning\x: macro:->{5}{10000}{0}{1}{1}{1113}{1}{0}{8}{1096}{8}{1}{1}
{17}{9}{1}{64}{8}{584}{65}{2}{1}{1177}{131}{8}{0}{10000}{1113}.
```

The first token is the number of steps, the second is N, then 0, 1, the GCD, M, 1, 0, the first quotient, the first remainder, the top left entry of the first matrix, the bottom left entry, and then these four things at each step until the end.

11.5 \xintTypesetEuclideanAlgorithm

This macro is just an example of how to organize the data returned by `\xintEuclideanAlgorithm`. Copy the source code to a new macro and modify it to what is needed.

```
\xintTypesetEuclideanAlgorithm {123456789012345}{9876543210321}
123456789012345 = 12 × 9876543210321 + 4938270488493
9876543210321 = 2 × 4938270488493 + 2233335
4938270488493 = 2211164 × 2233335 + 536553
2233335 = 4 × 536553 + 87123
536553 = 6 × 87123 + 13815
87123 = 6 × 13815 + 4233
13815 = 3 × 4233 + 1116
4233 = 3 × 1116 + 885
1116 = 1 × 885 + 231
885 = 3 × 231 + 192
231 = 1 × 192 + 39
192 = 4 × 39 + 36
39 = 1 × 36 + 3
36 = 12 × 3 + 0
```

11.6 \xintTypesetBezoutAlgorithm

This macro is just an example of how to organize the data returned by `\xintBezoutAlgorithm`. Copy the source code to a new macro and modify it to what is needed.

```
\xintTypesetBezoutAlgorithm {10000}{1113}
10000 = 8 × 1113 + 1096
8 = 8 × 1 + 0
1 = 8 × 0 + 1
1113 = 1 × 1096 + 17
9 = 1 × 8 + 1
1 = 1 × 1 + 0
1096 = 64 × 17 + 8
584 = 64 × 9 + 8
65 = 64 × 1 + 1
```

```

17 = 2 × 8 + 1
1177 = 2 × 584 + 9
131 = 2 × 65 + 1
8 = 8 × 1 + 0
10000 = 8 × 1177 + 584
1113 = 8 × 131 + 65
131 × 10000 − 1177 × 1113 = −1

```

12 Package **xint** implementation

The commenting of the macros is currently (2013/04/04) very sparse. Some comments may be left-overs from previous versions of the macro, with parameters in another order for example.

12.1 Catcodes, ε-TeX detection, reload detection

The method for package identification and reload detection is copied verbatim from the packages by HEIKO OBERDIEK.

The method for catcodes was also inspired by these packages, we proceed slightly differently.

```

1 \begingroup\catcode61\catcode48\catcode32=10\relax%
2   \catcode13=5      % ^^M
3   \endlinechar=13 %
4   \catcode123=1    % {
5   \catcode125=2    % }
6   \catcode64=11    % @
7   \catcode35=6     % #
8   \catcode44=12    % ,
9   \catcode45=12    % -
10  \catcode46=12    % .
11  \catcode58=12    % :
12 \expandafter\let\expandafter\x\csname ver@xint.sty\endcsname
13 \expandafter
14   \ifx\csname PackageInfo\endcsname\relax
15     \def\y#1#2{\immediate\write-1{Package #1 Info: #2.}}%
16   \else
17     \def\y#1#2{\PackageInfo{#1}{#2}}%
18   \fi
19 \expandafter
20 \ifx\csname numexpr\endcsname\relax
21   \y{xint}{\numexpr not available, aborting input}%
22   \aftergroup\endinput
23 \else
24   \ifx\x\relax % plain-TeX, first loading
25   \else
26     \def\empty {}%
27     \ifx\x\empty % LaTeX, first loading,
28       % variable is initialized, but \ProvidesPackage not yet seen
29     \else

```

```

30      \y{xint}{I was already loaded, aborting input}%
31      \aftergroup\endinput
32      \fi
33      \fi
34 \fi
35 \def\ChangeCatcodesIfInputNotAborted
36 {%
37     \endgroup
38     \edef\xint@restorecatcodes{\endinput
39     {%
40         \catcode47=\the\catcode47  % /
41         \catcode41=\the\catcode41  % )
42         \catcode40=\the\catcode40  % (
43         \catcode42=\the\catcode42  % *
44         \catcode43=\the\catcode43  % +
45         \catcode62=\the\catcode62  % >
46         \catcode60=\the\catcode60  % <
47         \catcode58=\the\catcode58  % :
48         \catcode46=\the\catcode46  % .
49         \catcode45=\the\catcode45  % -
50         \catcode44=\the\catcode44  % ,
51         \catcode35=\the\catcode35  % #
52         \catcode64=\the\catcode64  % @@
53         \catcode125=\the\catcode125 % }
54         \catcode123=\the\catcode123 % {
55         \endlinechar=\the\endlinechar
56         \catcode13=\the\catcode13  % ^M
57         \catcode32=\the\catcode32  %
58         \catcode61=\the\catcode61  % =
59         \noexpand\endinput
60     }%
61     \def\xint@setcatcodes
62     {%
63         \catcode61=12  % =
64         \catcode32=10  % space
65         \catcode13=5  % ^M
66         \endlinechar=13 %
67         \catcode123=1  % {
68         \catcode125=2  % }
69         \catcode64=11  % @@
70         \catcode35=6  % #
71         \catcode44=12  % ,
72         \catcode45=12  % -
73         \catcode46=12  % .
74         \catcode58=11  % : (made letter for error cs)
75         \catcode60=12  % <
76         \catcode62=12  % >
77         \catcode43=12  % +
78         \catcode42=12  % *
79         \catcode40=12  % (
80         \catcode41=12  % )
81         \catcode47=12  % /

```

```

82      }%
83      \XINT@setcatcodes
84  }%
85 \ChangeCatcodesIfInputNotAborted

```

12.2 Package identification

Copied verbatim from HEIKO OBERDIEK's packages.

```

86 \begingroup
87  \catcode91=12 % [
88  \catcode93=12 % ]
89  \catcode58=12 % : (does not really matter, was letter)
90  \expandafter\ifx\csname ProvidesPackage\endcsname\relax
91    \def\x#1#2#3[#4]{\endgroup
92      \immediate\write-1{Package: #3 #4}%
93      \xdef#1[#4]%
94    }%
95  \else
96    \def\x#1#2[#3]{\endgroup
97      #2[{#3}]%
98      \ifx#1\undefined
99        \xdef#1{#3}%
100       \fi
101      \ifx#1\relax
102        \xdef#1{#3}%
103       \fi
104     }%
105   \fi
106 \expandafter\x\csname ver@xint.sty\endcsname
107 \ProvidesPackage{xint}%
108 [2013/04/04 v1.01 Expandable operations on long numbers (jfB)]%

```

12.3 Token management macros

```

109 \def\xint@gobble      #1{}%
110 \def\xint@gobble@one  #1{}%
111 \def\xint@gobble@two  #1#2{}%
112 \def\xint@gobble@three #1#2#3{}%
113 \def\xint@gobble@four  #1#2#3#4{}%
114 \def\xint@gobble@five  #1#2#3#4#5{}%
115 \def\xint@gobble@six  #1#2#3#4#5#6{}%
116 \def\xint@gobble@seven #1#2#3#4#5#6#7{}%
117 \def\xint@gobble@eight #1#2#3#4#5#6#7#8{}%
118 \def\xint@secondoftwo #1#2{#2}%
119 \def\xint@firstoftwo@andstop #1#2{ #1}%
120 \def\xint@secondoftwo@andstop #1#2{ #2}%
121 \def\xint@exchangetwo@keepbraces #1#2{[#2]{#1}}%
122 \def\xint@exchangetwo@keepbraces@andstop #1#2{ {#2}{#1}}%
123 \def\xint@xpxp@andstop {\expandafter\expandafter\expandafter\space }%
124 \def\xint@r    #1\R {}%
125 \def\xint@w    #1\W {}%
126 \def\xint@z    #1\Z {}%

```

```

127 \def\xint@zero  #10{ }%
128 \def\xint@one   #11{ }%
129 \def\xint@minus #1-{ }%
130 \def\xint@relax #1\relax { }%
131 \def\xint@quatrezeros #10000{ }%
132 \def\xint@bracedundef {\xint@undef }%
133 \def\xint@UDzerofork      #10\dummy  #2#3\xint@UDkrof {#2}%
134 \def\xint@UDsignfork     #1-\dummy  #2#3\xint@UDkrof {#2}%
135 \def\xint@UDzerosfork    #10\dummy  #2#3\xint@UDkrof {#2}%
136 \def\xint@UDonezerofork  #110\dummy #2#3\xint@UDkrof {#2}%
137 \def\xint@UDzerominusfork #10-\dummy #2#3\xint@UDkrof {#2}%
138 \def\xint@UDsignsfork   #1--\dummy #2#3\xint@UDkrof {#2}%
139 \def\xint@afterfi #1#2\fi {\fi #1}%

```

12.4 **\xintRev**, **\xintReverseOrder**

\xintRev: fait la double expansion, vérifie le signe
\xintReverseOrder: ne fait PAS la double expansion, ne regarde PAS le signe.

```

140 \def\xintRev {\romannumeral0\xintrev }%
141 \def\xintrev #1%
142 {%
143     \expandafter\expandafter\expandafter
144     \xint@rev
145     \expandafter\expandafter\expandafter
146     {#1}%
147 }%
148 \def\xint@rev #1%
149 {%
150     \XINT@rev@fork #1\Z
151 }%
152 \def\XINT@rev@fork #1#2%
153 {%
154     \xint@UDsignfork
155     #1\dummy \XINT@rev@negative
156     -\dummy \XINT@rev@nonnegative
157     \xint@UDkrof
158     #1#2%
159 }%
160 \def\XINT@rev@negative #1#2\Z
161 {%
162     \expandafter
163     \space
164     \expandafter
165     -%
166     \romannumeral0\XINT@rev {#2}%
167 }%
168 \def\XINT@rev@nonnegative #1\Z
169 {%
170     \XINT@rev {#1}%
171 }%
172 \def\XINT@Rev {\romannumeral0\XINT@rev }%
173 \let\xintReverseOrder \XINT@Rev

```

```

174 \def\XINT@rev #1%
175 {%
176     \XINT@rord@main {}#1%
177     \xint@UNDEF
178     \xint@undef\xint@undef\xint@undef\xint@undef
179     \xint@undef\xint@undef\xint@undef\xint@undef
180     \xint@UNDEF
181 }%
182 \def\XINT@rord@main #1#2#3#4#5#6#7#8#9%
183 {%
184     \XINT@strip@undef #9\XINT@rord@cleanup\xint@undef
185     \XINT@rord@main {}#9#8#7#6#5#4#3#2#1}%
186 }%
187 \def\XINT@rord@cleanup\xint@undef\XINT@rord@main #1#2\xint@UNDEF
188 {%
189     \expandafter\space\XINT@strip@UNDEF #1%
190 }%
191 \def\XINT@strip@undef #1\xint@undef {}%
192 \def\XINT@strip@UNDEF #1\xint@UNDEF {}%

```

12.5 \XINT@RQ

cette macro renverse et ajoute le nombre minimal de zéros à la fin pour que la longueur soit alors multiple de 4
`romannumeral0\XINT@RQ {}<le truc à renverser>\R\R\R\R\R\R\R\R\Z

```

193 \def\XINT@RQ #1#2#3#4#5#6#7#8#9%
194 {%
195     \xint@r #9\XINT@RQ@end\R
196     \XINT@RQ {}#9#8#7#6#5#4#3#2#1}%
197 }%
198 \def\XINT@RQ@end\R\XINT@RQ #1#2\Z
199 {%
200     \XINT@RQ@end@ #1\Z
201 }%
202 \def\XINT@RQ@end@ #1#2#3#4#5#6#7#8%
203 {%
204     \xint@r #8\XINT@RQ@end@viii
205     #7\XINT@RQ@end@vii
206     #6\XINT@RQ@end@vi
207     #5\XINT@RQ@end@v
208     #4\XINT@RQ@end@iv
209     #3\XINT@RQ@end@iii
210     #2\XINT@RQ@end@ii
211     \R\XINT@RQ@end@i
212     \Z #2#3#4#5#6#7#8%
213 }%
214 \def\XINT@RQ@end@viii #1\Z #2#3#4#5#6#7#8#9\Z { #9}%
215 \def\XINT@RQ@end@vii #1\Z #2#3#4#5#6#7#8#9\Z { #8#9000}%
216 \def\XINT@RQ@end@vi #1\Z #2#3#4#5#6#7#8#9\Z { #7#8#900}%
217 \def\XINT@RQ@end@v #1\Z #2#3#4#5#6#7#8#9\Z { #6#7#8#90}%
218 \def\XINT@RQ@end@iv #1\Z #2#3#4#5#6#7#8#9\Z { #5#6#7#8#9}%
219 \def\XINT@RQ@end@iii #1\Z #2#3#4#5#6#7#8#9\Z { #4#5#6#7#8#9000}%

```

```
220 \def\xINT@RQ@end@ii #1\Z #2#3#4#5#6#7#8#9\Z { #3#4#5#6#7#8#900}%
221 \def\xINT@RQ@end@i \Z #1#2#3#4#5#6#7#8\Z { #1#2#3#4#5#6#7#80}%
```

12.6 **\XINT@cuZ**

```
222 \def\xint@cleanupzeros@andstop #1#2#3#4%
223 {\expandafter
224     \space
225     \the\numexpr #1#2#3#4\relax
226 }%
227 \def\xint@cleanupzeros@nospace #1#2#3#4%
228 {%
229     \the\numexpr #1#2#3#4\relax
230 }%
231 \def\xINT@Rev@andcleanupzeros #1%
232 {%
233     \romannumeral0\expandafter
234         \xint@cleanupzeros@andstop
235     \romannumeral0\xINT@rord@main {}#1%
236     \xint@UNDEF
237         \xint@undef\xint@undef\xint@undef\xint@undef\xint@undef
238         \xint@undef\xint@undef\xint@undef\xint@undef\xint@undef
239     \xint@UNDEF
240 }%
routine CleanUpZeros. Utilisée en particulier par la
soustraction.
INPUT: longueur **multiple de 4** (--- ATTENTION)
OUTPUT: on a retiré tous les leading zéros, on n'est **plus*
nécessairement de longueur 4n
Délimiteur pour @main: \W\W\W\W\W\W\W\Z avec SEPT \W

241 \def\xINT@cuZ #1%
242 {%
243     \XINT@cuZ@loop #1\W\W\W\W\W\W\W\Z%
244 }%
245 \def\xINT@cuZ@loop #1#2#3#4#5#6#7#8%
246 {%
247     \xint@w #8\xint@cuZ@enda\W
248     \xint@z #8\xint@cuZ@endb\Z
249     \XINT@cuZ@checka {#1#2#3#4#5#6#7#8}%
250 }%
251 \def\xint@cuZ@enda #1\xINT@cuZ@checka #2%
252 {%
253     \xint@cuZ@endaa #2%
254 }%
255 \def\xint@cuZ@endaa #1#2#3#4#5\Z
256 {%
257     \expandafter\space\the\numexpr #1#2#3#4\relax
258 }%
259 \def\xint@cuZ@endb\Z\xINT@cuZ@checka #1{ 0}%
260 \def\xINT@cuZ@checka #1{%
261 {%
262     \expandafter \XINT@cuZ@checkb \the\numexpr #1\relax
```

```
263 }%
264 \def\xINT@cuz@checkb #1%
265 {%
266   \xint@zero #1\xint@cuz@backtoloop 0\xINT@cuz@stop #1%
267 }%
268 \def\xINT@cuz@stop #1\W #2\Z{ #1}%
269 \def\xint@cuz@backtoloop 0\xINT@cuz@stop 0{\xINT@cuz@loop }%
```

12.7 \xintNum

```

270 \def\xintNum {\romannumeral0\xintnum }%
271 \def\xintnum #1%
272 {%
273   \expandafter\expandafter\expandafter
274     \XINT@num
275   \expandafter\expandafter\expandafter
276   {#1}%
277 }%
278 \def\XINT@Num {\romannumeral0\XINT@num }%
279 \def\XINT@num #1{\XINT@num@loop #1\R\R\R\R\R\R\R\R\Z }%
280 \def\XINT@num@loop #1#2#3#4#5#6#7#8%
281 {%
282   \xint@r #8\XINT@num@end\R\XINT@num@NumEight #1#2#3#4#5#6#7#8%
283 }%
284 \def\XINT@num@end\R\XINT@num@NumEight #1\R #2\Z
285 {%
286   \expandafter\space\the\numexpr #1+0\relax
287 }%
288 \def\XINT@num@NumEight #1#2#3#4#5#6#7#8%
289 {%
290   \ifnum \numexpr #1#2#3#4#5#6#7#8+0\relax = 0
291     \xint@afterfi {\expandafter\XINT@num@keepsign@a
292                   \the\numexpr #1#2#3#4#5#6#7#81\relax}%
293   \else
294     \xint@afterfi {\expandafter\XINT@num@finish
295                   \the\numexpr #1#2#3#4#5#6#7#8\relax}%
296   \fi
297 }%
298 \def\XINT@num@keepsign@a #1%
299 {%
300   \xint@one#1\XINT@num@gobacktoloop 1\XINT@num@keepsign@b
301 }%
302 \def\XINT@num@gobacktoloop 1\XINT@num@keepsign@b {\XINT@num@loop }%
303 \def\XINT@num@keepsign@b #1{\XINT@num@loop -}%
304 \def\XINT@num@finish #1\R #2\Z { #1}%

```

12.8 \xintLen, \xintLength

\xintLen -> fait la double expansion, ne compte PAS le signe


```

356     {\expandafter\space\the\numexpr #2-2\relax}%
357 \def\xint@length@end@i    #1\xint@length@end@viii #2%
358     {\expandafter\space\the\numexpr #2-1\relax}%

```

12.9 **\xintAssign**, **\xintAssignArray**, **\xintDigitsOf**

```

\xintAssign {a}{b}..{z}\to\A\B...\Z,
\xintAssignArray {a}{b}..{z}\to\U
version 1.01 corrects an oversight in 1.0 related to the value of
\escapechar at the time of using \xintAssignArray or \xintRelaxArray

359 \def\xintAssign #1\to
360 {%
361     \expandafter\expandafter\expandafter
362     \XINT@assign@a #1{} \to
363 }%
364 \def\xint@assign@a #1% attention to the # at the beginning of next line
365 #{%
366     \def\xint@temp {#1}%
367     \ifx\empty\xint@temp
368         \expandafter\XINT@assign@b
369     \else
370         \expandafter\XINT@assign@B
371     \fi
372 }%
373 \def\xint@assign@b #1#2\to #3%
374 {%
375     \edef #3{#1}\def\xint@temp {#2}%
376     \ifx\empty\xint@temp
377         \else
378             \xint@afterfi{\XINT@assign@a #2\to }%
379     \fi
380 }%
381 \def\xint@assign@B #1\to #2%
382 {%
383     \edef #2{\xint@temp}%
384 }%
385 \def\xintRelaxArray #1%
386 {%
387     \edef\xint@restoreescapechar {\escapechar\the\escapechar\relax}%
388     \escapechar -1
389     \edef\xint@arrayname {\string #1}%
390     \XINT@restoreescapechar
391     \expandafter\let\expandafter\xint@temp
392         \csname\xint@arrayname 0\endcsname
393     \count 255 0
394     \loop
395         \global\expandafter\let
396             \csname\xint@arrayname\the\count255\endcsname\relax
397         \ifnum \count 255 < \xint@temp
398             \advance\count 255 1
399         \repeat
400         \global\expandafter\let\csname\xint@arrayname 00\endcsname\relax

```

```

401     \global\let #1\relax
402 }%
403 \def\xintAssignArray #1\to #2%
404 {%
405     \edef\XINT@restoreescapechar {\escapechar\the\escapechar\relax}%
406     \escapechar -1
407     \edef\xint@arrayname {\string #1}%
408     \XINT@restoreescapechar
409     \count 255 0
410         \expandafter\expandafter\expandafter
411         \XINT@assignarray@loop #1\xint@undef
412         \csname\xint@arrayname 00\endcsname
413         \csname\xint@arrayname 0\endcsname
414         {\xint@arrayname}%
415     #2%
416 }%
417 \def\XINT@assignarray@loop #1%
418 {%
419     \def\xint@temp {\#1}%
420     \ifx\xint@bracedundef\xint@temp
421         \edef\xint@temp{\the\count 255 }%
422         \expandafter\let\csname\xint@arrayname0\endcsname\xint@temp
423         \expandafter\XINT@assignarray@end
424     \else
425         \advance\count 255 1
426         \expandafter\edef
427             \csname\xint@arrayname\the\count 255\endcsname{\xint@temp}%
428         \expandafter\XINT@assignarray@loop
429     \fi
430 }%
431 \def\XINT@assignarray@end {\expandafter\XINT@assignarray@@end }%
432 \def\XINT@assignarray@@end #1%
433 {%
434     \expandafter\XINT@assignarray@@@end\expandafter #1%
435 }%
436 \def\XINT@assignarray@@@end #1#2#3%
437 {%
438     \expandafter\XINT@assignarray@@@end
439     \expandafter #1\expandafter #2\expandafter{#3}%
440 }%
441 \def\XINT@assignarray@@@@end #1#2#3#4%
442 {%
443     \def #4##1%
444     {\romannumeral0%
445         \expandafter\expandafter\expandafter
446         ##1%
447         \expandafter\expandafter\expandafter
448         {##1}%
449     }%
450     \def #1##1%
451     {%
452         \ifnum ##1< 0

```

```

453         \xint@afterfi {\xintError:ArrayListIndexIsNegative
454                         \expandafter\space 0}%
455     \else
456         \xint@afterfi {%
457             \ifnum ##1> #2
458                 \xint@afterfi {\xintError:ArrayListIndexBeyondLimit
459                             \expandafter\space 0}%
460             \else
461                 \xint@afterfi
462                 {\expandafter\expandafter\expandafter
463                     \space\csname #3##1\endcsname}%
464             \fi}%
465         \fi
466     }%
467 }%
468 \let\xintDigitsOf\xintAssignArray

```

12.10 \xintSgn

```

469 \def\xintSgn {\romannumeral0\xintsgn }%
470 \def\xintsgn #1%
471 {%
472     \expandafter\expandafter\expandafter
473     \XINT@sgn #1\Z%
474 }%
475 \def\XINT@Sgn #1{\romannumeral0\XINT@sgn #1\Z }%
476 \def\XINT@sgn #1%
477 {%
478     \xint@xpxp@andstop
479     \xint@UDzerominusfork
480     #1-\dummy {\expandafter0} zero
481     0#1\dummy {\expandafter-\expandafter1} n\'egatif
482     0-\dummy {\expandafter1} positif
483     \xint@UDkrof
484     \xint@z
485 }%

```

12.11 \xintOpp

```

486 \def\xintOpp {\romannumeral0\xintopp }%
487 \def\xintopp #1%
488 {%
489     \expandafter\expandafter\expandafter
490     \XINT@opp #1%
491 }%
492 \def\XINT@Opp #1{\romannumeral0\XINT@opp #1}%
493 \def\XINT@opp #1%
494 {%
495     \expandafter\space
496     \xint@UDzerominusfork
497     #1-\dummy 0% zero
498     0#1\dummy {}% negative
499     0-\dummy {-#1}% positive

```

```

500     \xint@UDkrof
501 }%

```

12.12 \xintAbs

```

502 \def\xintAbs {\romannumeral0\xintabs }%
503 \def\xintabs #1%
504 {%
505     \expandafter\expandafter\expandafter
506     \XINT@abs #1%
507 }%
508 \def\XINT@Abs {\romannumeral0\XINT@abs }%
509 \def\XINT@abs #1%
510 {%
511     \xint@UDsignfork
512     #1\dummy \space
513     -\dummy { #1}%
514     \xint@UDkrof
515 }%
-----
```

ARITHMETIC OPERATIONS: ADDITION, SUBTRACTION, SUMS,
MULTIPLICATION, PRODUCTS, FACTORIAL, POWERS, EUCLIDEAN DIVISION.

12.13 \xintAdd

```

516 \def\xintAdd {\romannumeral0\xintadd }%
517 \def\xintadd #1%
518 {%
519     \expandafter\expandafter\expandafter
520     \xint@add
521     \expandafter\expandafter\expandafter
522     {#1}%
523 }%
524 \def\xint@add #1#2%
525 {%
526     \expandafter\expandafter\expandafter
527     \XINT@add@fork #2\Z #1\Z
528 }%
529 \def\XINT@Add #1#2{\romannumeral0\XINT@add@fork #2\Z #1\Z }%
530 \def\XINT@add #1#2{\XINT@add@fork #2\Z #1\Z }%
      ADDITION
      Ici #1#2 vient du *deuxième* argument de \xintAdd
      et #3#4 donc du *premier* [algo plus efficace lorsque
      le premier est plus long que le second]

531 \def\XINT@add@fork #1#2\Z #3#4\Z
532 {%
533     \xint@UDzerofork
534     #1\dummy \XINT@add@secondiszero
535     #3\dummy \XINT@add@firstiszero
536     0\dummy
537     {\xint@UDsignsfork

```

```

538      #1#3\dummy \XINT@add@minusminus          % #1 = #3 = -
539      #1-\dummy \XINT@add@minusplus          % #1 =
540      #3-\dummy \XINT@add@plusminus          % #3 =
541      --\dummy \XINT@add@plusplus
542      \xint@UDkrof }%
543      \xint@UDkrof
544      {#2}{#4}#1#3%
545 }%
546 \def\xint@add@secondiszero #1#2#3#4{ #4#2}%
547 \def\xint@add@firstiszero #1#2#3#4{ #3#1}%

#1 vient du *deuxième* et #2 vient du *premier*

548 \def\xint@add@minusminus #1#2#3#4%
549 {%
550     \expandafter\space\expandafter-%
551     \romannumeral0\xint@add@pre {#2}{#1}%
552 }%
553 \def\xint@add@minusplus #1#2#3#4%
554 {%
555     \XINT@sub@pre {#4#2}{#1}%
556 }%
557 \def\xint@add@plusminus #1#2#3#4%
558 {%
559     \XINT@sub@pre {#3#1}{#2}%
560 }%
561 \def\xint@add@plusplus #1#2#3#4%
562 {%
563     \XINT@add@pre {#4#2}{#3#1}%
564 }%
565 \def\xint@add@pre #1%
566 {%
567     \expandafter\xint@add@@pre\expandafter{%
568     \romannumeral0\xint@RQ {}#1\R\R\R\R\R\R\R\R\Z
569     }%
570 }%
571 \def\xint@add@@pre #1#2%
572 {%
573     \expandafter\xint@add@A
574         \expandafter0\expandafter{\expandafter}%
575     \romannumeral0\xint@RQ {}#2\R\R\R\R\R\R\R\R\Z
576         \W\X\Y\Z #1\W\X\Y\Z
577 }%

ADDITION \XINT@add@A
INPUT:
\romannumeral0\xint@add@A <N1>\W\X\Y\Z <N2>\W\X\Y\Z
avec: N1 et N2 sur **4n**, et **renversés**, et le plus long ne
doit pas se terminer par 0000. [Donc on peut avoir 0000 comme
input si l'autre est >0 et ne se termine pas en 0000 bien sûr].
OUTPUT:
La somme N1+N2, *PAS* sur 4n, dans l'ordre *normal*, et *sans
leading zeros*

```

12 Package **xint** implementation

La procédure est plus rapide lorsque la longueur de N2 est supérieure à celle de N1

```

578 \def\xint@add@A #1#2#3#4#5#6%
579 {%
580     \xint@w
581     #3\xint@add@az
582     \W\xint@add@AB #1{#3#4#5#6}{#2}%
583 }%
      1er nombre fini.

584 \def\xint@add@az\W\xint@add@AB #1#2%
585 {%
586     \XINT@add@AC@checkcarry #1%
587 }%
      ici #2 est prévu pour l'addition, mais attention il devra être renversé pour
      \numexpr. #3 = résultat partiel. #4 = chiffres qui restent

588 \def\xint@add@AB #1#2#3#4\W\X\Y\Z #5#6#7#8%
589 {%
590     \xint@w
591     #5\xint@add@bz
592     \W\xint@add@ABE #1#2{#8#7#6#5}{#3}#4\W\X\Y\Z
593 }%
594 \def\xint@add@ABE #1#2#3#4#5#6%
595 {\expandafter
596     \XINT@add@ABEA\the\numexpr #1+10#5#4#3#2+#6\relax.%
597 }%
598 \def\xint@add@ABEA #1#2#3.#4%
599 {%
600     \XINT@add@A #2{#3#4}%
601 }%
      ici le deuxième nombre est fini
      #6 part à la poubelle, #2#3#4#5 est le #2 dans \XINT@add@AB
      on ne vérifie pas la retenue cette fois, mais les fois suivantes

602 \def\xint@add@bz\W\xint@add@ABE #1#2#3#4#5#6%
603 {\expandafter
604     \XINT@add@CC\the\numexpr #1+10#5#4#3#2\relax.%
605 }%
606 \def\xint@add@CC #1#2#3.#4%
607 {%
608     \XINT@add@AC@checkcarry #2{#3#4}%
       on va examiner et \eliminer #2
609 }%
      retenue plus chiffres qui restent de l'un des deux nombres.
      #2 = résultat partiel
      #3#4#5#6 = summand, avec plus significatif à droite

```

```

610 \def\xINT@add@AC@checkcarry #1%
611 {%
612     \xint@zero #1\xint@add@AC@nocarry 0\xINT@add@C
613 }%
614 \def\xint@add@AC@nocarry 0\xINT@add@C #1#2\W\X\Y\Z
615 {%
616     \expandafter
617     \xint@cleanupzeros@andstop
618     \romannumeral0%
619     \XINT@rord@main {}#2%
620     \xint@UNDEF
621         \xint@undef\xint@undef\xint@undef\xint@undef
622         \xint@undef\xint@undef\xint@undef\xint@undef
623         \xint@UNDEF
624     #1%
625 }%
626 \def\xINT@add@C #1#2#3#4#5%
627 {%
628     \xint@w
629     #2\xint@add@cz
630     \W\xINT@add@CD {#5#4#3#2}{#1}%
631 }%
632 \def\xINT@add@CD #1%
633 {\expandafter
634     \XINT@add@CC\the\numexpr 1+10#1\relax.%
635 }%
636 \def\xint@add@cz\W\xINT@add@CD #1#2{ 1#2}%

```

12.14 \xintSub

```

637 \def\xintSub {\romannumeral0\xintsub }%
638 \def\xintsub #1%
639 {%
640     \expandafter\expandafter\expandafter
641             \xint@sub
642     \expandafter\expandafter\expandafter
643             {#1}%
644 }%
645 \def\xint@sub #1#2%
646 {%
647     \expandafter\expandafter\expandafter
648             \XINT@sub@fork #2\Z #1\Z
649 }%
650 \def\xINT@Sub #1#2{\romannumeral0\xINT@sub@fork #2\Z #1\Z }%
651 \def\xINT@sub #1#2{\XINT@sub@fork #2\Z #1\Z }%
SOUSTRACTION
#3#4-#1#2
#3#4 vient du *premier*
#1#2 vient du *second*

652 \def\xINT@sub@fork #1#2\Z #3#4\Z
653 {%
654     \xint@UDsignsfork

```

```

655 #1#3\dummy \XINT@sub@minusminus
656 #1-\dummy \XINT@sub@minusplus % attention, #3=0 possible
657 #3-\dummy \XINT@sub@plusminus % attention, #1=0 possible
658 --\dummy {\xint@UDzerofork
659 #1\dummy \XINT@sub@secondiszero
660 #3\dummy \XINT@sub@firstiszero
661 0\dummy \XINT@sub@plusplus
662 \xint@UDkrof }%
663 \xint@UDkrof
664 {#2}{#4}#1#3%
665 }%
666 \def\xint@sub@secondiszero #1#2#3#4{ #4#2}%
667 \def\xint@sub@firstiszero #1#2#3#4{ -#3#1}%
668 \def\xint@sub@plusplus #1#2#3#4%
669 {%
670 \XINT@sub@pre {#4#2}{#3#1}%
671 }%
672 \def\xint@sub@minusminus #1#2#3#4%
673 {%
674 \XINT@sub@pre {#1}{#2}%
675 }%
676 \def\xint@sub@minusplus #1#2#3#4%
677 {%
678 \xint@zero #4\xint@sub@mp0\xint@add@pre {#4#2}{#1}%
679 }%
680 \def\xint@sub@mp0\xint@add@pre #1#2{ #2}%
681 \def\xint@sub@plusminus #1#2#3#4%
682 {%
683 \xint@zero #3\xint@sub@pm0\expandafter\space\expandafter-%
684 \romannumeral0\xint@add@pre {#2}{#3#1}%
685 }%
686 \def\xint@sub@pm #1\xint@add@pre #2#3{ -#2}%
687 \def\xint@sub@pre #1%
688 {%
689 \expandafter\xint@sub@@pre\expandafter{%
690 \romannumeral0\xint@RQ { }#1\R\R\R\R\R\R\R\R\Z
691 }%
692 }%
693 \def\xint@sub@@pre #1#2%
694 {%
695 \expandafter\xint@sub@A
696 \expandafter1\expandafter{\expandafter}%
697 \romannumeral0\xint@RQ { }#2\R\R\R\R\R\R\R\R\Z
698 \W\X\Y\Z #1 \W\X\Y\Z
699 }%

```

\romannumeral0\xint@subA 1{ }<N1>\W\X\Y\Z<N2>\W\X\Y\Z
 N1 et N2 sont présentés à l'envers ET ON A RAJOUTÉ DES ZÉROS
 POUR QUE LEURS LONGUEURS À CHACUN SOIENT MULTIPLES DE 4, MAIS
 AUCUN NE SE TERMINE EN 0000
 output: N2 - N1
 Elle donne le résultat dans le **bon ordre**, avec le bon signe,
 et sans zéros superflus.

```

700 \def\xint@sub@a #1#2#3\W\X\Y\Z #4#5#6#7%
701 {%
702     \xint@w
703     #4\xint@sub@az
704     \W\xint@sub@b #1{#4#5#6#7}{#2}#3\W\X\Y\Z
705 }%
706 \def\xint@sub@b #1#2#3#4#5#6#7%
707 {%
708     \xint@w
709     #4\xint@sub@bz
710     \W\xint@sub@onestep #1#2{#7#6#5#4}{#3}%
711 }%
d'abord la branche principale
#6 = 4 chiffres de N1, plus significatif en *premier*,
#2#3#4#5 chiffres de N2, plus significatif en *dernier*
On veut N2 - N1.

712 \def\xint@sub@onestep #1#2#3#4#5#6%
713 {\expandafter
714     \xint@sub@backtoA\the\numexpr 11#5#4#3#2-#6+#1-1\relax.%
715 }%
ON PRODUIT LE RÉSULTAT DANS LE BON ORDRE

716 \def\xint@sub@backtoA #1#2#3.#4%
717 {%
718     \xint@sub@a #2{#3#4}%
719 }%
720 \def\xint@sub@bz
721     \W\xint@sub@onestep #1#2#3#4#5#6#7%
722 {%
723     \xint@UDzerofork
724         #1\dummy \xint@sub@c % une retenue
725         0\dummy \xint@sub@d % pas de retenue
726     \xint@UDkrof
727     {#7}#2#3#4#5%
728 }%
729 \def\xint@sub@D #1#2\W\X\Y\Z
730 {%
731     \expandafter
732     \xint@cleanupzeros@andstop
733     \romannumeral0%
734     \xint@rord@main {}#2%
735     \xint@UNDEF
736         \xint@undef\xint@undef\xint@undef\xint@undef
737         \xint@undef\xint@undef\xint@undef\xint@undef
738     \xint@UNDEF
739     #1%
740 }%
741 \def\xint@sub@c #1#2#3#4#5%
742 {%
743     \xint@w

```

```

744      #2\xint@sub@cz
745      \W\XINT@sub@AC@onestep {#5#4#3#2}{#1}%
746 }%
747 \def\XINT@sub@AC@onestep #1%
748 {\expandafter
749   \XINT@sub@backtoC\the\numexpr 11#1-1\relax.%
750 }%
751 \def\XINT@sub@backtoC #1#2#3.#4%
752 {%
753   \XINT@sub@AC@checkcarry #2{#3#4}% la retenue va \^etre examin\'ee
754 }%
755 \def\XINT@sub@AC@checkcarry #1%
756 {%
757   \xint@one #1\xint@sub@AC@nocarry 1\XINT@sub@C
758 }%
759 \def\xint@sub@AC@nocarry 1\XINT@sub@C #1#2\W\X\Y\Z
760 {%
761   \expandafter
762   \XINT@cuz@loop
763   \romannumeral0%
764   \XINT@rord@main {}#2%
765   \xint@UNDEF
766     \xint@undef\xint@undef\xint@undef\xint@undef
767     \xint@undef\xint@undef\xint@undef\xint@undef
768     \xint@UNDEF
769   #1\W\W\W\W\W\W\W\Z
770 }%
771 \def\xint@sub@cz\W\XINT@sub@AC@onestep #1%
772 {%
773   \XINT@cuz
774 }%
775 \def\xint@sub@az\W\XINT@sub@B #1#2#3#4#5#6#7%
776 {%
777   \xint@w
778   #4\xint@sub@ez
779   \W\XINT@sub@Eenter #1{#3}#4#5#6#7%
780 }%

```

le premier nombre continue, le résultat sera < 0.

```

781 \def\XINT@sub@Eenter #1#2%
782 {%
783   \expandafter
784   \XINT@sub@E\expandafter1\expandafter{\expandafter}%
785   \romannumeral0%
786   \XINT@rord@main {}#2%
787   \xint@UNDEF
788     \xint@undef\xint@undef\xint@undef\xint@undef
789     \xint@undef\xint@undef\xint@undef\xint@undef
790     \xint@UNDEF
791   \W\X\Y\Z #1%
792 }%
793 \def\XINT@sub@E #1#2#3#4#5#6%

```

```

794 {%
795     \xint@w #3\xint@sub@F\W\XINT@sub@Eonestep
796     #1{#6#5#4#3}{#2}%
797 }%
798 \def\XINT@sub@Eonestep #1#2%
799 {\expandafter
800     \XINT@sub@backtoE\the\numexpr 110000-#2+#1-1\relax.%
801 }%
802 \def\XINT@sub@backtoE #1#2#3.#4%
803 {%
804     \XINT@sub@E #2{#3#4}%
805 }%
806 \def\xint@sub@F\W\XINT@sub@Eonestep #1#2#3#4%
807 {%
808     \xint@UDonezerofork
809     #4#1\dummy {\XINT@sub@Fdec 0}% soustraire 1. Et faire signe -
810     #1#4\dummy {\XINT@sub@Finc 1}% additionner 1. Et faire signe -
811     10\dummy \XINT@sub@DD % terminer. Mais avec signe -
812     \xint@UDkrof
813     {#3}%
814 }%
815 \def\XINT@sub@DD
816 {\expandafter\space\expandafter-\romannumeral0\XINT@sub@D }%
817 \def\XINT@sub@Fdec #1#2#3#4#5#6%
818 {%
819     \xint@w
820     #3\xint@sub@Fdec@finish\W\XINT@sub@Fdec@onestep
821     #1{#6#5#4#3}{#2}%
822 }%
823 \def\XINT@sub@Fdec@onestep #1#2%
824 {\expandafter
825     \XINT@sub@backtoFdec\the\numexpr 11#2+#1-1\relax.%
826 }%
827 \def\XINT@sub@backtoFdec #1#2#3.#4%
828 {%
829     \XINT@sub@Fdec #2{#3#4}%
830 }%
831 \def\xint@sub@Fdec@finish\W\XINT@sub@Fdec@onestep #1#2%
832 {%
833     \expandafter\space\expandafter-\romannumeral0\XINT@cuz
834 }%
835 \def\XINT@sub@Finc #1#2#3#4#5#6%
836 {%
837     \xint@w
838     #3\xint@sub@Finc@finish\W\XINT@sub@Finc@onestep
839     #1{#6#5#4#3}{#2}%
840 }%
841 \def\XINT@sub@Finc@onestep #1#2%
842 {\expandafter
843     \XINT@sub@backtoFinc\the\numexpr 10#2+#1\relax.%
844 }%
845 \def\XINT@sub@backtoFinc #1#2#3.#4%

```

```

846 {%
847     \XINT@sub@Finc #2{#3#4}%
848 }%
849 \def\xint@sub@Finc@finish\W\XINT@sub@Finc@onestep #1#2#3%
850 {%
851     \xint@UDzerofork
852     #1\dummy {\expandafter\space\expandafter-%
853             \xint@cleanupzeros@nospace}%
854     0\dummy {-1}%
855     \xint@UDkrof
856     #3%
857 }%
858 \def\xint@sub@ez\W\XINT@sub@Enter #1%
859 {%
860     \xint@UDzerofork
861     #1\dummy \XINT@sub@K % il y a une retenue
862     0\dummy \XINT@sub@L % pas de retenue
863     \xint@UDkrof
864 }%
865 \def\XINT@sub@L #1\W\X\Y\Z
866     {\XINT@cuz@loop #1\W\W\W\W\W\W\W\Z }%
867 \def\XINT@sub@K #1%
868 {%
869     \expandafter
870     \XINT@sub@KK\expandafter1\expandafter{\expandafter}%
871     \romannumeral0%
872     \XINT@rord@main {}#1%
873     \xint@UNDEF
874     \xint@undef\xint@undef\xint@undef\xint@undef
875     \xint@undef\xint@undef\xint@undef\xint@undef
876     \xint@UNDEF
877 }%
878 \def\XINT@sub@KK #1#2#3#4#5#6%
879 {%
880     \xint@w
881     #3\xint@sub@KK@finish\W\XINT@sub@KK@onestep
882     #1{#6#5#4#3}{#2}%
883 }%
884 \def\XINT@sub@KK@onestep #1#2%
885 {\expandafter
886     \XINT@sub@backtoKK\the\numexpr 110000-#2+#1-1\relax.%
887 }%
888 \def\XINT@sub@backtoKK #1#2#3.#4%
889 {%
890     \XINT@sub@KK #2{#3#4}%
891 }%
892 \def\xint@sub@KK@finish\W\XINT@sub@KK@onestep #1#2#3%
893 {%
894     \expandafter\space\expandafter-\romannumeral
895     0\XINT@cuz@loop #3\W\W\W\W\W\W\W\Z
896 }%

```

12.15 \xintCmp


```

989 \def\xint@cmp@L #1{\xint@OneIfPositive@main #1}%
990 \def\xint@OneIfPositive #1%
991 {%
992     \xint@OneIfPositive@main #1\W\X\Y\Z%
993 }%
994 \def\xint@OneIfPositive@main #1#2#3#4%
995 {%
996     \xint@z #4\xint@OneIfPositive@terminated\Z\xint@OneIfPositive@onestep
997     #1#2#3#4%
998 }%
999 \def\xint@OneIfPositive@terminated\Z\xint@OneIfPositive@onestep\W\X\Y\Z { 0}%
1000 \def\xint@OneIfPositive@onestep #1#2#3#4%
1001 {%
1002     \expandafter
1003     \xint@OneIfPositive@check
1004     \the\numexpr #1#2#3#4\relax
1005 }%
1006 \def\xint@OneIfPositive@check #1%
1007 {%
1008     \xint@zero
1009     #1\xint@OneIfPositive@backtomain 0\xint@OneIfPositive@finish #1%
1010 }%
1011 \def\xint@OneIfPositive@finish #1\W\X\Y\Z{ 1}%
1012 \def\xint@OneIfPositive@backtomain 0\xint@OneIfPositive@finish 0%
1013             {\xint@OneIfPositive@main }%

```

12.16 **\xintGeq**

PLUS GRAND OU ÉGAL
attention compare les **valeurs absolues**

```

1014 \def\xintGeq {\romannumeral0\xintgeq }%
1015 \def\xintgeq #1%
1016 {%
1017     \expandafter\expandafter\expandafter
1018     \xint@geq
1019     \expandafter\expandafter\expandafter
1020     {#1}%
1021 }%
1022 \def\xint@geq #1#2%
1023 {\expandafter\expandafter\expandafter
1024     \XINT@geq@fork #2\Z #1\Z
1025 }%
1026 \def\xint@Geq #1#2{\romannumeral0\xint@geq@fork #2\Z #1\Z }%

```

PLUS GRAND OU ÉGAL
ATTENTION, TESTE les VALEURS ABSOLUES

```

1027 \def\xint@geq@fork #1#2\Z #3#4\Z
1028 {%
1029     \xint@UDzerofork
1030     #1\dummy \XINT@geq@secondiszero % |#1#2|=0
1031     #3\dummy \XINT@geq@firstiszero % |#1#2|>0

```

PLUS GRAND OU ÉGAL
N1 et N2 sont présentés à l'envers ET ON A RAJOUTÉ DES ZÉROS
POUR QUE LEURS LONGUEURS À CHACUN SOIENT MULTIPLES DE 4, MAIS
AUCUN NE SE TERMINE EN 0000
routine appelée via
`\romannumeral0\XINT@geq@A 1{}<N1>\W\X\Y\Z<N2>\W\X\Y\Z`
ATTENTION RENVOIE 1 SI N1 < N2 ou N1 = N2 et 0 si N1 > N2

```

1064 \def\XINT@geq@a #1#2#3\W\x\Y\Z #4#5#6#7%
1065 {%
1066     \xint@w
1067     #4\xint@geq@az
1068     \W\XINT@geq@b #1{#4#5#6#7}{#2}#3\W\x\Y\Z
1069 }%
1070 \def\XINT@geq@b #1#2#3#4#5#6#7%
1071 {%
1072     \xint@w
1073     #4\xint@geq@bz
1074     \W\XINT@geq@onestep #1#2{#7#6#5#4}{#3}%
1075 }%

```

```

1076 \def\xINT@geq@onestep #1#2#3#4#5#6%
1077 {\expandafter
1078   \XINT@geq@backtoA\the\numexpr 11#5#4#3#2-#6+#1-1\relax.%%
1079 }%
1080 \def\xINT@geq@backtoA #1#2#3.#4%
1081 {%
1082   \XINT@geq@A #2{#3#4}%
1083 }%
1084 \def\xint@geq@bz\W\xINT@geq@onestep #1\W\X\Y\Z { 1}%
1085 \def\xint@geq@az\W\xINT@geq@B #1#2#3#4#5#6#7%
1086 {%
1087   \xint@w
1088   #4\xint@geq@ez
1089   \W\xINT@geq@Eenter #1%
1090 }%
1091 \def\xINT@geq@Eenter #1\W\X\Y\Z { 0}%
1092 \def\xint@geq@ez\W\xINT@geq@Eenter #1%
1093 {%
1094   \xint@UDzerofork
1095   #1\dummy { 0} % il y a une retenue
1096   0\dummy { 1} % pas de retenue
1097   \xint@UDkrof
1098 }%

```

12.17 \xintMax

```

1099 \def\xintMax {\romannumeral0\xintmax }%
1100 \def\xintmax #1%
1101 {%
1102   \expandafter\expandafter\expandafter
1103   \xint@max
1104   \expandafter\expandafter\expandafter
1105   {#1}%
1106 }%
1107 \def\xint@max #1#2%
1108 {%
1109   \expandafter\expandafter\expandafter
1110   \XINT@max@fork #2\Z #1\Z
1111 }%
1112 \def\xINT@Max #1#2{\romannumeral0\xINT@max@fork #2\Z #1\Z }%
  #3#4 vient du *premier*
  #1#2 vient du *second*
1113 \def\xINT@max@fork #1#2\Z #3#4\Z
1114 {%
1115   \xint@UDsignsfork
1116   #1#3\dummy \XINT@max@minusminus % A < 0, B < 0
1117   #1-\dummy \XINT@max@minusplus % B < 0, A >= 0
1118   #3-\dummy \XINT@max@plusminus % A < 0, B >= 0
1119   --\dummy {\xint@UDzerosfork
1120     #1#3\dummy \XINT@max@zerozero % A = B = 0
1121     #10\dummy \XINT@max@zeroplus % B = 0, A > 0
1122     #30\dummy \XINT@max@pluszero % A = 0, B > 0

```

```

1123          @0\dummy \XINT@max@plusplus % A, B > 0
1124          \xint@UDkrof }%
1125          \xint@UDkrof
1126          {#2}{#4}#1#3%
1127 }%

A = #4#2, B = #3#1

1128 \def\XINT@max@zerozero #1#2#3#4{ 0}%
1129 \def\XINT@max@zeroplus #1#2#3#4{ #4#2}%
1130 \def\XINT@max@pluszero #1#2#3#4{ #3#1}%
1131 \def\XINT@max@minusplus #1#2#3#4{ #4#2}%
1132 \def\XINT@max@plusminus #1#2#3#4{ #3#1}%
1133 \def\XINT@max@plusplus #1#2#3#4%
1134 {%
1135     \ifodd\XINT@Geq {#4#2}{#3#1}
1136         \xint@afterfi { #4#2}%
1137     \else
1138         \xint@afterfi { #3#1}%
1139     \fi
1140 }%

#3=-, #4=-, #1 = |B| = -B, #2 = |A| = -A

1141 \def\XINT@max@minusminus #1#2#3#4%
1142 {%
1143     \ifodd\XINT@Geq {#1}{#2}
1144         \xint@afterfi { -#2}%
1145     \else
1146         \xint@afterfi { -#1}%
1147     \fi
1148 }%

```

12.18 \xintMin

```

1149 \def\xintMin {\romannumeral0\xintmin }%
1150 \def\xintmin #1%
1151 {%
1152     \expandafter\expandafter\expandafter
1153         \xint@min
1154     \expandafter\expandafter\expandafter
1155         {#1}%
1156 }%
1157 \def\xint@min #1#2%
1158 {%
1159     \expandafter\expandafter\expandafter
1160         \XINT@min@fork #2\Z #1\Z
1161 }%
1162 \def\XINT@Min #1#2{\romannumeral0\xint@min@fork #2\Z #1\Z }%
#3#4 vient du *premier*
#1#2 vient du *second*

```

12 Package **xint** implementation

```

1163 \def\XINT@min@fork #1#2\Z #3#4\Z
1164 {%
1165     \xint@UDsignsfork
1166         #1#3\dummy \XINT@min@minusminus % A < 0, B < 0
1167         #1-\dummy \XINT@min@minusplus % B < 0, A >= 0
1168         #3-\dummy \XINT@min@plusminus % A < 0, B >= 0
1169         --\dummy {\xint@UDzerosfork
1170             #1#3\dummy \XINT@min@zerozero % A = B = 0
1171             #10\dummy \XINT@min@zeroplus % B = 0, A > 0
1172             #30\dummy \XINT@min@pluszero % A = 0, B > 0
1173             00\dummy \XINT@min@plusplus % A, B > 0
1174             \xint@UDkrof }%
1175     \xint@UDkrof
1176     {#2}{#4}{#1#3%
1177 }%
A = #4#2, B = #3#1

1178 \def\XINT@min@zerozero #1#2#3#4{ 0}%
1179 \def\XINT@min@zeroplus #1#2#3#4{ 0}%
1180 \def\XINT@min@pluszero #1#2#3#4{ 0}%
1181 \def\XINT@min@minusplus #1#2#3#4{ #3#1}%
1182 \def\XINT@min@plusminus #1#2#3#4{ #4#2}%
1183 \def\XINT@min@plusplus #1#2#3#4%
1184 {%
1185     \ifodd\XINT@Geq {#4#2}{#3#1}
1186         \xint@afterfi { #3#1}%
1187     \else
1188         \xint@afterfi { #4#2}%
1189     \fi
1190 }%
#3=-, #4=-, #1 = |B| = -B, #2 = |A| = -A

1191 \def\XINT@min@minusminus #1#2#3#4%
1192 {%
1193     \ifodd\XINT@Geq {#1}{#2}
1194         \xint@afterfi { -#1}%
1195     \else
1196         \xint@afterfi { -#2}%
1197     \fi
1198 }%

```

12.19 \xintSum, \xintSumExpr

```

\xintSum {{a}{b}}...{z}
\xintSumExpr {a}{b}}...{z}\relax

1199 \def\XINT@psum #1%
1200 {%
1201     \romannumeral0\XINT@psum@checkifemptysum #1\Z
1202 }%
1203 \def\XINT@psum@checkifemptysum #1%
1204 {%

```



```

1257 }%
1258 \def\xINT@sum@skipzeroinput #1\xint@UDkrof #2\Z #3#4%
1259 {%
1260     \XINT@sum@xpxpnext {#3}{#4}%
1261 }%
1262 \def\xINT@sum@pushpos #1#2\Z #3#4%
1263 {%
1264     \XINT@sum@xpxpnext {#3{#1#2}}{#4}%
1265 }%
1266 \def\xINT@sum@pushneg #1#2\Z #3#4%
1267 {%
1268     \XINT@sum@xpxpnext {#3}{#4{#2}}%
1269 }%
1270 \def\xINT@sum@xpxpnext #1#2#3%
1271 {%
1272     \expandafter\expandafter\expandafter
1273     \XINT@sum@checkiffinished #3\Z {#1}{#2}%
1274 }%
1275 \def\xINT@sum@checkiffinished #1%
1276 {%
1277     \xint@relax #1\xINT@sum@end\relax
1278     \XINT@sum@checksign #1%
1279 }%
1280 \def\xINT@sum@end\relax\xINT@sum@checksign\relax #1\Z #2#3%
1281     {\xintsub{#2\relax}{#3\relax}}%
1282 \def\xINT@sum@A #1#2#3#4#5#6%
1283 {%
1284     \xint@w
1285     #3\xint@sum@az
1286     \W\xINT@sum@B #1{#3#4#5#6}{#2}%
1287 }%
1288 \def\xint@sum@az\W\xINT@sum@B #1#2%
1289 {%
1290     \XINT@sum@AC@checkcarry #1%
1291 }%
1292 \def\xINT@sum@B #1#2#3#4\W\X\Y\Z #5#6#7#8%
1293 {%
1294     \xint@w
1295     #5\xint@sum@bz
1296     \W\xINT@sum@E #1#2{#8#7#6#5}{#3}{#4}\W\X\Y\Z
1297 }%
1298 \def\xINT@sum@E #1#2#3#4#5#6%
1299 {\expandafter
1300     \XINT@sum@ABEA\the\numexpr #1+10#5#4#3#2+#6\relax
1301 }%
1302 \def\xINT@sum@ABEA #1#2#3#4#5#6#7%
1303 {%
1304     \XINT@sum@A #2{#7#6#5#4#3}%
1305 }%
1306 \def\xint@sum@bz\W\xINT@sum@E #1#2#3#4#5#6%
1307 {\expandafter
1308     \XINT@sum@CC\the\numexpr #1+10#5#4#3#2\relax

```

```

1309 }%
1310 \def\XINT@sum@CC #1#2#3#4#5#6#7%
1311 {%
1312     \XINT@sum@AC@checkcarry #2{#7#6#5#4#3}%
1313 }%
1314 \def\XINT@sum@AC@checkcarry #1%
1315 {%
1316     \xint@zero #1\xint@sum@AC@nocarry 0\XINT@sum@C
1317 }%
1318 \def\xint@sum@AC@nocarry 0\XINT@sum@C #1#2\W\X\Y\Z { #1#2}%
1319 \def\XINT@sum@C #1#2#3#4#5%
1320 {%
1321     \xint@w
1322     #2\xint@sum@cz
1323     \W\XINT@sum@D {#5#4#3#2}{#1}%
1324 }%
1325 \def\XINT@sum@D #1%
1326 {\expandafter
1327     \XINT@sum@CC\the\numexpr 1+10#1\relax
1328 }%
1329 \def\xint@sum@cz\W\XINT@sum@D #1#2{ #21000}%

```

12.20 \xintMul

```

1330 \def\xintMul {\romannumeral0\xintmul }%
1331 \def\xintmul #1%
1332 {%
1333     \expandafter\expandafter\expandafter
1334         \xint@mul
1335     \expandafter\expandafter\expandafter
1336         {#1}%
1337 }%
1338 \def\xint@mul #1#2%
1339 {\expandafter\expandafter\expandafter
1340     \XINT@mul@fork #2\Z #1\Z
1341 }%
1342 \def\XINT@Mul #1#2{\romannumeral0\XINT@mul@fork #2\Z #1\Z }%
    MULTIPLICATION
    Ici #1#2 = 2e input et #3#4 = 1er input
    Algorithme plus efficace pour #3#4 plus long que #1#2

1343 \def\XINT@mul@fork #1#2\Z #3#4\Z
1344 {%
1345     \xint@UDzerofork
1346     #1\dummy \XINT@mul@zero
1347     #3\dummy \XINT@mul@zero
1348     0\dummy
1349     {\xint@UDsignsfork
1350         #1#3\dummy \XINT@mul@minusminus % #1 = #3 = -
1351         #1-\dummy \XINT@mul@minusplus % #1 = -
1352         #3-\dummy \XINT@mul@plusminus % #3 = -
1353         --\dummy \XINT@mul@plusplus
1354     \xint@UDkrof }%

```

```

1355      \xint@UDkrof
1356      {#2}{#4}#1#3%
1357 }%
1358 \def\xint@mul@zero #1#2#3#4{ 0}%

```

Dans ce qui suit #3#1 vient du #1#2 initial correspondant au
 ** 2e ** input.

```

1359 \def\xint@mul@minusminus #1#2#3#4%
1360 {%
1361     \expandafter
1362         \XINT@mul@enter\romannumeral0%
1363         \XINT@RQ {}#2\R\R\R\R\R\R\R\Z
1364         \W\X\Y\Z #1\W\X\Y\Z
1365 }%
1366 \def\xint@mul@minusplus #1#2#3#4%
1367 {%
1368     \expandafter\space\expandafter-
1369     \romannumeral0\expandafter
1370         \XINT@mul@enter\romannumeral0%
1371         \XINT@RQ {}#4#2\R\R\R\R\R\R\R\Z
1372         \W\X\Y\Z #1\W\X\Y\Z
1373 }%
1374 \def\xint@mul@plusminus #1#2#3#4%
1375 {%
1376     \expandafter\space\expandafter-
1377     \romannumeral0\expandafter
1378         \XINT@mul@enter\romannumeral0%
1379         \XINT@RQ {}#2\R\R\R\R\R\R\R\Z
1380         \W\X\Y\Z #3#1\W\X\Y\Z
1381 }%

```

Ici #3#1 correspond au **2e input** celui censé être
 psychologiquement plus petit

```

1382 \def\xint@mul@plusplus #1#2#3#4%
1383 {%
1384     \expandafter
1385         \XINT@mul@enter\romannumeral0%
1386         \XINT@RQ {}#4#2\R\R\R\R\R\R\R\Z
1387         \W\X\Y\Z #3#1\W\X\Y\Z
1388 }%
1389 \def\xint@mul@add@A #1#2#3#4#5#6%
1390 {%
1391     \xint@w
1392     #3\xint@mul@add@az
1393     \W\XINT@mul@add@AB #1{#3#4#5#6}{#2}%
1394 }%
1395 \def\xint@mul@add@az\W\xint@mul@add@AB #1#2%
1396 {%
1397     \XINT@mul@add@AC@checkcarry #1%
1398 }%
1399 \def\xint@mul@add@AB #1#2#3#4\W\X\Y\Z #5#6#7#8%

```

```

1400 {%
1401     \XINT@mul@add@ABE #1#2{#8#7#6#5}{#3}#4\W\X\Y\Z
1402 }%
1403 \def\XINT@mul@add@ABE #1#2#3#4#5#6%
1404 {\expandafter
1405     \XINT@mul@add@ABEA\the\numexpr #1+10#5#4#3#2+#6\relax.%}
1406 }%
1407 \def\XINT@mul@add@ABEA #1#2#3.#4%
1408 {%
1409     \XINT@mul@add@A #2{#3#4}%
1410 }%
1411 \def\XINT@mul@add@AC@checkcarry #1%
1412 {%
1413     \xint@zero #1\xint@mul@add@AC@nocarry 0\XINT@mul@add@C
1414 }%
1415 \def\xint@mul@add@AC@nocarry 0\XINT@mul@add@C #1#2\W\X\Y\Z
1416 {%
1417     \expandafter
1418     \xint@cleanupzeros@andstop
1419     \romannumberal0%
1420     \XINT@rord@main {}#2%
1421     \xint@UNDEF
1422         \xint@undef\xint@undef\xint@undef\xint@undef
1423         \xint@undef\xint@undef\xint@undef\xint@undef
1424         \xint@UNDEF
1425     #1%
1426 }%
1427 \def\XINT@mul@add@C #1#2#3#4#5%
1428 {%
1429     \xint@w
1430     #5\xint@mul@add@cw
1431     #4\xint@mul@add@cx
1432     #3\xint@mul@add@cy
1433     #2\xint@mul@add@cz
1434     \W\XINT@mul@add@CD {}#5#4#3#2}{}#1}%
1435 }%
1436 \def\XINT@mul@add@CD #1%
1437 {\expandafter
1438     \XINT@mul@add@CC\the\numexpr 1+10#1\relax.%}
1439 }%
1440 \def\XINT@mul@add@CC #1#2#3.#4%
1441 {%
1442     \XINT@mul@add@AC@checkcarry #2{#3#4}%
1443 }%
1444 \def\xint@mul@add@cw
1445     #1\xint@mul@add@cx
1446     #2\xint@mul@add@cy
1447     #3\xint@mul@add@cz
1448     \W\XINT@mul@add@CD
1449 {\expandafter
1450     \XINT@mul@add@CDw\the\numexpr 1+#1#2#3\relax.%}
1451 }%

```

```

1452 \def\xint@mul@add@CDw #1.#2#3\X\Y\Z
1453 {%
1454     \xint@mul@add@end #1#3%
1455 }%
1456 \def\xint@mul@add@cx
1457     #1\xint@mul@add@cy
1458     #2\xint@mul@add@cz
1459     \W\xint@mul@add@CD
1460 {\expandafter
1461     \xint@mul@add@CDx\the\numexpr 1+#1#2\relax.%
1462 }%
1463 \def\xint@mul@add@CDx #1.#2#3\Y\Z
1464 {%
1465     \xint@mul@add@end #1#3%
1466 }%
1467 \def\xint@mul@add@cy
1468     #1\xint@mul@add@cz
1469     \W\xint@mul@add@CD
1470 {\expandafter
1471     \xint@mul@add@CDy\the\numexpr 1+#1\relax.%
1472 }%
1473 \def\xint@mul@add@CDy #1.#2#3\Z
1474 {%
1475     \xint@mul@add@end #1#3%
1476 }%
1477 \def\xint@mul@add@cz\W\xint@mul@add@CD #1#2#3{\xint@mul@add@end #1#3}%
1478 \def\xint@mul@add@end #1#2#3#4#5%
1479 {\expandafter\space
1480   \the\numexpr #1#2#3#4#5\relax
1481 }%
1482 \def\xint@mul@Ar #1#2#3#4#5#6%
1483 {%
1484     \xint@z #6\xint@mul@br\Z\xint@mul@Br #1{#6#5#4#3}{#2}%
1485 }%
1486 \def\xint@mul@br\Z\xint@mul@Br #1#2%
1487 {%
1488     \xint@sum@AC@checkcarry #1%
1489 }%
1490 \def\xint@mul@Br #1#2#3#4\W\X\Y\Z #5#6#7#8%
1491 {\expandafter
1492     \xint@mul@ABEAr\the\numexpr #1+10#2+#8#7#6#5\relax.{#3}#4\W\X\Y\Z
1493 }%
1494 \def\xint@mul@ABEAr #1#2#3#4#5#6.#7%
1495 {%
1496     \xint@mul@Ar #2{#7#6#5#4#3}%
1497 }%

```

Mr renvoie le résultat ***à l'envers***, sur ***4n chiffres***

```

1498 \def\xint@mul@Mr #1%
1499 {%
1500     \expandafter
1501     \xint@mul@Mr@checkifzeroorone

```

```

1502     \expandafter{\the\numexpr #1\relax}%
1503 }%
1504 \def\xint@mul@Mr@checkifzeroorone #1%
1505 {%
1506     \ifcase #1
1507         \expandafter\xint@mul@Mr@zero
1508     \or
1509         \expandafter\xint@mul@Mr@one
1510     \else
1511         \expandafter\xint@mul@Nr
1512     \fi
1513     {0000}{}{#1}%
1514 }%
1515 \def\xint@mul@Mr@zero #1\Z\Z\Z\Z { 0000}%
1516 \def\xint@mul@Mr@one #1#2#3#4\Z\Z\Z\Z { #4}%
1517 \def\xint@mul@Nr #1#2#3#4#5#6#7%
1518 {%
1519     \xint@z #4\xint@mul@pr\Z\xint@mul@Pr {#1}{#3}{#7#6#5#4}{#2}{#3}%
1520 }%
1521 \def\xint@mul@Pr #1#2#3%
1522 {\expandafter
1523     \xint@mul@Lr\the\numexpr 10000#1+#2*#3\relax
1524 }%
1525 \def\xint@mul@Lr 1#1#2#3#4#5#6#7#8#9%
1526 {%
1527     \xint@mul@Nr {#1#2#3#4}{#9#8#7#6#5}%
1528 }%
1529 \def\xint@mul@pr\Z\xint@mul@Pr #1#2#3#4#5%
1530 {%
1531     \xint@quatrezeros #1\xint@mul@Mr@end@nocarry 0000\xint@mul@Mr@end@carry
1532     #1{#4}%
1533 }%
1534 \def\xint@mul@Mr@end@nocarry 0000\xint@mul@Mr@end@carry 0000#1{ #1}%
1535 \def\xint@mul@Mr@end@carry #1#2#3#4#5{ #5#4#3#2#1}%
1536 \def\xint@mul@M #1%
1537 {\expandafter
1538     \xint@mul@M@checkifzeroorone
1539     \expandafter{\the\numexpr #1\relax}%
1540 }%
1541 \def\xint@mul@M@checkifzeroorone #1%
1542 {%
1543     \ifcase #1
1544         \expandafter\xint@mul@M@zero
1545     \or
1546         \expandafter\xint@mul@M@one
1547     \else
1548         \expandafter\xint@mul@N
1549     \fi
1550     {0000}{}{#1}%
1551 }%
1552 \def\xint@mul@M@zero #1\Z\Z\Z\Z { 0}%
1553 \def\xint@mul@M@one #1#2#3#4\Z\Z\Z\Z {%

```

```

1554     \expandafter
1555         \xint@cleanupzeros@andstop
1556     \romannumeral0\XINT@rev{#4}%
1557 }%
1558 \def\XINT@mul@N #1#2#3#4#5#6#7%
1559 {%
1560     \xint@z #4\xint@mul@p\Z\XINT@mul@P {#1}{#3}{#7#6#5#4}{#2}{#3}%
1561 }%
1562 \def\XINT@mul@P #1#2#3%
1563 {\expandafter
1564     \XINT@mul@L\the\numexpr 10000#1+#2*#3\relax
1565 }%
1566 \def\XINT@mul@L 1#1#2#3#4#5#6#7#8#9%
1567 {%
1568     \XINT@mul@N {#1#2#3#4}{#5#6#7#8#9}%
1569 }%
1570 \def\xint@mul@p\Z\XINT@mul@P #1#2#3#4#5%
1571 {%
1572     \XINT@mul@M@end #1#4%
1573 }%
1574 \def\XINT@mul@M@end #1#2#3#4#5#6#7#8%
1575 {\expandafter\space
1576     \the\numexpr #1#2#3#4#5#6#7#8\relax
1577 }%

```

Routine de multiplication principale
délimiteur $\backslash W\backslash X\backslash Y\backslash Z$
Le résultat partiel est toujours maintenu avec significatif à
droite et il a un nombre multiple de 4 de chiffres
 $\romannumeral0\XINT@mul@enter <N1>\backslash W\backslash X\backslash Y\backslash Z <N2>\backslash W\backslash X\backslash Y\backslash Z$
avec N1: *renversé*, *longueur 4n* (zéros éventuellement ajoutés
au-delà du chiffre le plus significatif)
et N2 = dans l'ordre *normal*, et pas forcément longueur 4n,
et N2 est *non nul*.
pas de signes

```

1578 \def\XINT@mul@enter #1\W\X\Y\Z #2#3#4#5%
1579 {%
1580     \xint@w
1581     #5\xint@mul@enterw
1582     #4\xint@mul@enterx
1583     #3\xint@mul@entry
1584     #2\xint@mul@enterz
1585     \W\XINT@mul@start {#2#3#4#5}#1\W\X\Y\Z
1586 }%
1587 \def\xint@mul@enterw
1588     #1\xint@mul@enterx
1589     #2\xint@mul@entry
1590     #3\xint@mul@enterz
1591     \W\XINT@mul@start #4#5\W\X\Y\Z \X\Y\Z
1592 {%
1593     \XINT@mul@M {#3#2#1}#5\Z\Z\Z\Z
1594 }%

```

```

1595 \def\xint@mul@enterx
1596     #1\xint@mul@entery
1597     #2\xint@mul@enterz
1598     \W\XINT@mul@start  #3#4\W\X\Y\Z \Y\Z
1599 {%
1600     \XINT@mul@M {\#2#1}#4\Z\Z\Z\Z
1601 }%
1602 \def\xint@mul@entery
1603     #1\xint@mul@enterz
1604     \W\XINT@mul@start  #2#3\W\X\Y\Z \Z
1605 {%
1606     \XINT@mul@M {\#1}#3\Z\Z\Z\Z
1607 }%
1608 \def\XINT@mul@start #1#2\W\X\Y\Z
1609 {\expandafter
1610     \XINT@mul@main \expandafter
1611     {\romannumeral0\XINT@mul@Mr {\#1}#2\Z\Z\Z\Z}#2\W\X\Y\Z
1612 }%
1613 \def\XINT@mul@main #1#2\W\X\Y\Z #3#4#5#6%
1614 {%
1615     \xint@w
1616     #6\xint@mul@mainw
1617     #5\xint@mul@mainx
1618     #4\xint@mul@mainy
1619     #3\xint@mul@mainz
1620     \W\XINT@mul@compute {\#1}{#3#4#5#6}#2\W\X\Y\Z
1621 }%
1622 \def\XINT@mul@compute #1#2#3\W\X\Y\Z
1623 {\expandafter
1624     \XINT@mul@main \expandafter
1625     {\romannumeral0\expandafter
1626     \XINT@mul@Ar \expandafter{\expandafter{\expandafter}}%
1627     \romannumeral0\XINT@mul@Mr {\#2}#3\Z\Z\Z\Z \W\X\Y\Z 0000#1\W\X\Y\Z
1628     }#3\W\X\Y\Z
1629 }%
1630 \def\xint@mul@mainw
1631     #1\xint@mul@mainx
1632     #2\xint@mul@mainy
1633     #3\xint@mul@mainz
1634     \W\XINT@mul@compute #4#5#6\W\X\Y\Z \X\Y\Z
1635 {%
1636     \expandafter
1637     \XINT@mul@add@A \expandafter{\expandafter{\expandafter}}%
1638     \romannumeral0%
1639     \XINT@mul@Mr {\#3#2#1}#6\Z\Z\Z\Z
1640     \W\X\Y\Z 000#4\W\X\Y\Z
1641 }%
1642 \def\xint@mul@mainx
1643     #1\xint@mul@mainy
1644     #2\xint@mul@mainz
1645     \W\XINT@mul@compute #3#4#5\W\X\Y\Z \Y\Z
1646 {%

```

```

1647     \expandafter
1648     \XINT@mul@add@A \expandafter\expandafter{\expandafter}%
1649         \romannumeral0%
1650     \XINT@mul@Mr {\#2#1}#5\Z\Z\Z\Z
1651             \W\X\Y\Z 00#3\W\X\Y\Z
1652 }%
1653 \def\xint@mul@mainy
1654     #1\xint@mul@mainz
1655     \W\XINT@mul@compute #2#3#4\W\X\Y\Z \Z
1656 {%
1657     \expandafter
1658     \XINT@mul@add@A \expandafter\expandafter{\expandafter}%
1659         \romannumeral0%
1660     \XINT@mul@Mr {\#1}#4\Z\Z\Z\Z
1661             \W\X\Y\Z 0#2\W\X\Y\Z
1662 }%
1663 \def\xint@mul@mainz\W\XINT@mul@compute #1#2#3\W\X\Y\Z
1664 {%
1665     \expandafter
1666     \xint@cleanupzeros@andstop\romannumeral0\XINT@rev{\#1}%
1667 }%

```

12.21 \xintSqr

```

1668 \def\xintSqr {\romannumeral0\xintsqr }%
1669 \def\xintsqr #1%
1670 {%
1671     \expandafter\expandafter\expandafter
1672     \XINT@sqr
1673     \expandafter\expandafter\expandafter
1674     {\xintAbs{\#1}}% fait l'expansion de #1 et se d'ebarasse du signe
1675 }%
1676 \def\XINT@sqr #1%
1677 {\expandafter
1678     \XINT@mul@enter
1679         \romannumeral0%
1680     \XINT@RQ {}#1\R\R\R\R\R\R\R\R\R\R\Z
1681         \W\X\Y\Z #1\W\X\Y\Z
1682 }%

```

12.22 \xintPrd, \xintProductExpr

```

\xintPrd {{a}...{z}}
\xintProductExpr {a}...{z}\relax

1683 \def\XINT@posprod #1%
1684 {%
1685     \XINT@pprod@checkifempty #1\Z
1686 }%
1687 \def\XINT@pprod@checkifempty #1%
1688 {%
1689     \xint@relax #1\XINT@pprod@emptyproduct\relax
1690     \XINT@pprod@RQfirst #1%
1691 }%

```

```

1692 \def\XINT@pprod@emptyproduct #1\Z { 1}%
1693 \def\XINT@pprod@RQfirst #1\Z
1694 {%
1695     \expandafter\XINT@pprod@getnext\expandafter
1696     {\romannumeral0\XINT@RQ {}#1\R\R\R\R\R\R\R\R\Z}%
1697 }%
1698 \def\XINT@pprod@getnext #1#2%
1699 {%
1700     \XINT@pprod@checkiffinished #2\Z {#1}%
1701 }%
1702 \def\XINT@pprod@checkiffinished #1%
1703 {%
1704     \xint@relax #1\XINT@pprod@end\relax
1705     \XINT@pprod@compute #1%
1706 }%
1707 \def\XINT@pprod@compute #1\Z #2%
1708 {%
1709     \expandafter
1710         \XINT@pprod@getnext
1711     \expandafter
1712     {\romannumeral0\XINT@prod@mul@enter #2\W\X\Y\Z #1\W\X\Y\Z}%
1713 }%
1714 \def\XINT@pprod@end\relax\XINT@pprod@compute #1\Z #2%
1715 {%
1716     \expandafter
1717     \xint@cleanupzeros@andstop
1718     \romannumeral0\XINT@rev {#2}%
1719 }%
1720 \def\xintProductExpr {\romannumeral0\xintproductexpr }%
1721 \def\xintPrd {\romannumeral0\xintprd }%
1722 \def\xintprd #1%
1723 {%
1724     \expandafter\expandafter\expandafter
1725     \xintproductexpr #1\relax
1726 }%
1727 \def\xintproductexpr #1%
1728 {%
1729     \expandafter\expandafter\expandafter
1730     \XINT@prod@checkifempty #1\Z
1731 }%
1732 \def\XINT@prod@checkifempty #1%
1733 {%
1734     \xint@relax #1\XINT@prod@emptyproduct\relax
1735     \XINT@prod@checkfirstsign #1%
1736 }%
1737 \def\XINT@prod@emptyproduct #1\Z { 1}%
1738 \def\XINT@prod@checkfirstsign #1%
1739 {%
1740     \xint@zero #1\XINT@prod@returnzero0%
1741     \xint@UDsignfork
1742     #1\dummy \XINT@prod@firstisneg
1743     -\dummy \XINT@prod@firstispos

```



```

1796 {%
1797     \expandafter
1798         \xint@prod@cleanupzeros
1799     \romannumeral0\XINT@rev {\#2#3}%
1800 }%
1801 \def\xint@prod@cleanupzeros #1#2#3#4#5%
1802 {%
1803     \expandafter\space\the\numexpr (2*#1-1)*#2#3#4#5\relax
1804 }%


    MULTIPLICATION ET ADDITION POUR LES PRODUITS

1805 \def\XINT@prod@add@A #1#2#3#4#5#6%
1806 {%
1807     \xint@w
1808     #3\xint@prod@add@az
1809     \W\XINT@prod@add@AB #1{\#3#4#5#6}{#2}%
1810 }%
1811 \def\xint@prod@add@az\W\XINT@prod@add@AB #1#2%
1812 {%
1813     \XINT@prod@add@AC@checkcarry #1%
1814 }%
1815 \def\XINT@prod@add@AC@checkcarry #1%
1816 {%
1817     \xint@zero #1\xint@prod@add@AC@nocarry 0\XINT@prod@add@C
1818 }%
1819 \def\xint@prod@add@AC@nocarry 0\XINT@prod@add@C
1820 {%
1821     \XINT@prod@add@F
1822 }%
1823 \def\XINT@prod@add@AB #1#2#3#4\W\X\Y\Z #5#6#7#8%
1824 {%
1825     \XINT@prod@add@ABE #1#2{\#8#7#6#5}{#3}{#4}\W\X\Y\Z
1826 }%
1827 \def\XINT@prod@add@ABE #1#2#3#4#5#6%
1828 {\expandafter
1829     \XINT@prod@add@ABEA\the\numexpr #1+10#5#4#3#2+#6\relax
1830 }%
1831 \def\XINT@prod@add@ABEA #1#2#3#4#5#6#7%
1832 {%
1833     \XINT@prod@add@A #2{\#7#6#5#4#3}{#1}%
1834 }%
1835 \def\XINT@prod@add@C #1#2#3#4#5%
1836 {%
1837     \xint@w
1838     #5\xint@prod@add@cw
1839     #4\xint@prod@add@cx
1840     #3\xint@prod@add@cy
1841     #2\xint@prod@add@cz
1842     \W\XINT@prod@add@CD {#5#4#3#2}{#1}%
1843 }%
1844 \def\XINT@prod@add@CD #1%
1845 {\expandafter

```

```

1846      \XINT@prod@add@CC\the\numexpr 1+10#1\relax
1847 }%
1848 \def\XINT@prod@add@CC #1#2#3#4#5#6#7%
1849 {%
1850     \XINT@prod@add@AC@checkcarry #2{#7#6#5#4#3}%
1851 }%
1852 \def\xint@prod@add@cw
1853     #1\xint@prod@add@cx
1854     #2\xint@prod@add@cy
1855     #3\xint@prod@add@cz
1856     \W\XINT@prod@add@CD
1857 {\expandafter
1858     \XINT@prod@add@CDw\the\numexpr 1+10#1#2#3\relax
1859 }%
1860 \def\XINT@prod@add@CDw #1#2#3#4#5#6%
1861 {%
1862     \xint@quatrezeros #2#3#4#5\XINT@prod@add@endDw@zeros
1863             0000\XINT@prod@add@endDw #2#3#4#5%
1864 }%
1865 \def\XINT@prod@add@endDw@zeros 0000\XINT@prod@add@endDw 0000#1\X\Y\Z{ #1}%
1866 \def\XINT@prod@add@endDw #1#2#3#4#5\X\Y\Z{ #5#4#3#2#1}%
1867 \def\xint@prod@add@cx
1868     #1\xint@prod@add@cy
1869     #2\xint@prod@add@cz
1870     \W\XINT@prod@add@CD
1871 {\expandafter
1872     \XINT@prod@add@CDx\the\numexpr 1+100#1#2\relax
1873 }%
1874 \def\XINT@prod@add@CDx #1#2#3#4#5#6%
1875 {%
1876     \xint@quatrezeros #2#3#4#5\XINT@prod@add@endDx@zeros
1877             0000\XINT@prod@add@endDx #2#3#4#5%
1878 }%
1879 \def\XINT@prod@add@endDx@zeros 0000\XINT@prod@add@endDx 0000#1\Y\Z{ #1}%
1880 \def\XINT@prod@add@endDx #1#2#3#4#5\Y\Z{ #5#4#3#2#1}%
1881 \def\xint@prod@add@cy
1882     #1\xint@prod@add@cz
1883     \W\XINT@prod@add@CD
1884 {\expandafter
1885     \XINT@prod@add@CDy\the\numexpr 1+1000#1\relax
1886 }%
1887 \def\XINT@prod@add@CDy #1#2#3#4#5#6%
1888 {%
1889     \xint@quatrezeros #2#3#4#5\XINT@prod@add@endDy@zeros
1890             0000\XINT@prod@add@endDy #2#3#4#5%
1891 }%
1892 \def\XINT@prod@add@endDy@zeros 0000\XINT@prod@add@endDy 0000#1\Z{ #1}%
1893 \def\XINT@prod@add@endDy #1#2#3#4#5\Z{ #5#4#3#2#1}%
1894 \def\xint@prod@add@cz\W\XINT@prod@add@CD #1#2{ #21000}%
1895 \def\XINT@prod@add@F #1#2#3#4#5%
1896 {%
1897     \xint@w

```

```

1898      #5\xint@prod@add@Gw
1899      #4\xint@prod@add@Gx
1900      #3\xint@prod@add@Gy
1901      #2\xint@prod@add@Gz
1902      \W\XINT@prod@add@G {#2#3#4#5}{#1}%
1903 }%
1904 \def\XINT@prod@add@G #1#2%
1905 {%
1906     \XINT@prod@add@F {#2#1}%
1907 }%
1908 \def\xint@prod@add@Gw
1909     #1\xint@prod@add@Gx
1910     #2\xint@prod@add@Gy
1911     #3\xint@prod@add@Gz
1912     \W\XINT@prod@add@G #4%
1913 {%
1914     \xint@quatrezeros #3#2#10\XINT@prod@add@endGw@zeros
1915             0000\XINT@prod@add@endGw #3#2#10%
1916 }%
1917 \def\XINT@prod@add@endGw@zeros 0000\XINT@prod@add@endGw 0000#1\X\Y\Z{ #1}%
1918 \def\XINT@prod@add@endGw #1#2#3#4#5\X\Y\Z{ #5#1#2#3#4}%
1919 \def\xint@prod@add@Gx
1920     #1\xint@prod@add@Gy
1921     #2\xint@prod@add@Gz
1922     \W\XINT@prod@add@G #3%
1923 {%
1924     \xint@quatrezeros #2#100\XINT@prod@add@endGx@zeros
1925             0000\XINT@prod@add@endGx #2#100%
1926 }%
1927 \def\XINT@prod@add@endGx@zeros 0000\XINT@prod@add@endGx 0000#1\Y\Z{ #1}%
1928 \def\XINT@prod@add@endGx #1#2#3#4#5\Y\Z{ #5#1#2#3#4}%
1929 \def\xint@prod@add@Gy
1930     #1\xint@prod@add@Gz
1931     \W\XINT@prod@add@G #2%
1932 {%
1933     \xint@quatrezeros #1000\XINT@prod@add@endGy@zeros
1934             0000\XINT@prod@add@endGy #1000%
1935 }%
1936 \def\XINT@prod@add@endGy@zeros 0000\XINT@prod@add@endGy 0000#1\Z{ #1}%
1937 \def\XINT@prod@add@endGy #1#2#3#4#5\Z{ #5#1#2#3#4}%
1938 \def\xint@prod@add@Gz\W\XINT@prod@add@G #1#2{ #2}%

--- multiplication spéciale

1939 \def\XINT@prod@mul@enter #1\W\X\Y\Z #2#3#4#5%
1940 {%
1941     \xint@w
1942     #5\xint@prod@mul@enterw
1943     #4\xint@prod@mul@enterx
1944     #3\xint@prod@mul@entery
1945     #2\xint@prod@mul@enterz
1946     \W\XINT@prod@mul@start {#2#3#4#5}#1\W\X\Y\Z
1947 }%

```

```

1948 \def\xint@prod@mul@enterw
1949     #1\xint@prod@mul@enterx
1950     #2\xint@prod@mul@entery
1951     #3\xint@prod@mul@enterz
1952     \W\XINT@prod@mul@start #4#5\W\X\Y\Z \X\Y\Z
1953 {%
1954     \XINT@mul@Mr {#3#2#1}#5\Z\Z\Z\Z
1955 }%
1956 \def\xint@prod@mul@enterx
1957     #1\xint@prod@mul@entery
1958     #2\xint@prod@mul@enterz
1959     \W\XINT@prod@mul@start #3#4\W\X\Y\Z \Y\Z
1960 {%
1961     \XINT@mul@Mr {#2#1}#4\Z\Z\Z\Z
1962 }%
1963 \def\xint@prod@mul@entery
1964     #1\xint@prod@mul@enterz
1965     \W\XINT@prod@mul@start #2#3\W\X\Y\Z \Z
1966 {%
1967     \XINT@mul@Mr {#1}#3\Z\Z\Z\Z
1968 }%
1969 \def\XINT@prod@mul@start #1#2\W\X\Y\Z
1970 {\expandafter
1971     \XINT@prod@mul@main \expandafter
1972         {\romannumeral0%
1973             \XINT@mul@Mr {#1}#2\Z\Z\Z\Z
1974             }#2\W\X\Y\Z
1975 }%
1976 \def\XINT@prod@mul@main #1#2\W\X\Y\Z #3#4#5#6%
1977 {%
1978     \xint@w
1979     #6\xint@prod@mul@mainw
1980     #5\xint@prod@mul@mainx
1981     #4\xint@prod@mul@mainy
1982     #3\xint@prod@mul@mainz
1983     \W\XINT@prod@mul@compute {#1}{#3#4#5#6}#2\W\X\Y\Z
1984 }%
1985 \def\XINT@prod@mul@compute #1#2#3\W\X\Y\Z
1986 {\expandafter
1987     \XINT@prod@mul@main \expandafter
1988         {\romannumeral0\expandafter
1989             \XINT@mul@Ar \expandafter{\expandafter{\expandafter}%
1990             \romannumeral0\XINT@mul@Mr {#2}#3\Z\Z\Z\Z \W\X\Y\Z 0000#1\W\X\Y\Z
1991             }#3\W\X\Y\Z
1992 }%
1993 \def\xint@prod@mul@mainw
1994     #1\xint@prod@mul@mainx
1995     #2\xint@prod@mul@mainy
1996     #3\xint@prod@mul@mainz
1997     \W\XINT@prod@mul@compute #4#5#6\W\X\Y\Z \X\Y\Z
1998 {%
1999     \expandafter

```

```

2000  \XINT@prod@add@A \expandafter\expandafter{\expandafter}%
2001      \romannumeral0%
2002  \XINT@mul@Mr  {#3#2#1}#6\Z\Z\Z\Z
2003          \W\X\Y\Z 000#4\W\X\Y\Z
2004 }%
2005 \def\xint@prod@mul@mainx
2006     #1\xint@prod@mul@mainy
2007     #2\xint@prod@mul@mainz
2008     \W\XINT@prod@mul@compute #3#4#5\W\X\Y\Z \Y\Z
2009 {%
2010     \expandafter
2011     \XINT@prod@add@A \expandafter\expandafter{\expandafter}%
2012         \romannumeral0%
2013     \XINT@mul@Mr  {#2#1}#5\Z\Z\Z\Z
2014             \W\X\Y\Z 00#3\W\X\Y\Z
2015 }%
2016 \def\xint@prod@mul@mainy
2017     #1\xint@prod@mul@mainz
2018     \W\XINT@prod@mul@compute #2#3#4\W\X\Y\Z \Z
2019 {%
2020     \expandafter
2021     \XINT@prod@add@A \expandafter\expandafter{\expandafter}%
2022         \romannumeral0%
2023     \XINT@mul@Mr  {#1}#4\Z\Z\Z\Z
2024             \W\X\Y\Z 0#2\W\X\Y\Z
2025 }%
2026 \def\xint@prod@mul@mainz\W\XINT@prod@mul@compute #1#2#3\W\X\Y\Z
2027 { #1}%

```

12.23 \xintFac

```

2028 \def\xintFac {\romannumeral0\xintfac }%
2029 \def\xintfac #1%
2030 {%
2031     \expandafter\expandafter\expandafter
2032         \XINT@fac@fork
2033     \expandafter\expandafter\expandafter
2034         {#1}%
2035 }%
2036 \def\XINT@Fac {\romannumeral0\XINT@fac@fork }%
2037 \def\XINT@fac@fork #1%
2038 {%
2039     \ifcase\xintSgn {#1}
2040         \xint@afterfi{\expandafter\space\expandafter 1\xint@gobble }%
2041     \or
2042         \expandafter\XINT@fac@checklength
2043     \else
2044         \xint@afterfi{\xintError:FactorialOfNegativeNumber
2045             \expandafter\space\expandafter 1\xint@gobble }%
2046     \fi
2047     {#1}%
2048 }%

```

```

2049 \def\xINT@fac@checklength #1%
2050 {%
2051     \ifnum\xintLen {#1} > 9
2052         \xint@afterfi{\xintError:FactorialOfTooBigNumber
2053             \expandafter\space\expandafter 1\xint@gobble@three }%
2054     \else
2055         \expandafter\XINT@fac@loop
2056     \fi
2057     {1}{#1}{}}%
2058 }%
2059 \def\xINT@fac@loop #1#2#3%
2060 {%
2061     \ifnum #1<#2
2062         \expandafter
2063             \XINT@fac@loop
2064         \expandafter
2065             {\the\numexpr #1+1\expandafter }%
2066     \else
2067         \expandafter\xINT@fac@docomputation
2068     \fi
2069     {#2}{#3{#1}}%
2070 }%
2071 \def\xINT@fac@docomputation #1#2%
2072 {%
2073     \xINT@posprod #2\relax
2074 }%

```

12.24 \xintPow

```

2075 \def\xintPow {\romannumeral0\xintpow }%
2076 \def\xintpow #1%
2077 {%
2078     \expandafter\expandafter\expandafter
2079         \xint@pow
2080         #1\Z%
2081 }%
#1#2 = A

2082 \def\xint@pow #1#2\Z
2083 {%
2084     \xint@UDsignfork
2085         #1\dummy \XINT@pow@Aneg
2086         -\dummy \XINT@pow@Anonneg
2087     \xint@UDkrof
2088         #1{#2}%
2089 }%
2090 \def\xINT@pow@Aneg #1#2#3%
2091 {%
2092     \expandafter\expandafter\expandafter
2093         \XINT@pow@Aneg@
2094     \expandafter\expandafter\expandafter
2095         {#3}{#2}%
2096 }%

```

```

B = #1, xpxp déjà fait

2097 \def\xint@pow@Aneg@ #1%
2098 {%
2099   \ifcase\xint@Odd{#1}%
2100   \or \expandafter\xint@pow@Aneg@Bodd
2101   \fi
2102   \xint@pow@Anonneg@ {#1}%
2103 }%
2104 \def\xint@pow@Aneg@Bodd #1%
2105 {%
2106   \expandafter\xint@opp\romannumeral0\xint@pow@Anonneg@
2107 }%

B = #3, faire le xpxp

2108 \def\xint@pow@Anonneg #1#2#3%
2109 {%
2110   \expandafter\expandafter\expandafter
2111   \xint@pow@Anonneg@
2112   \expandafter\expandafter\expandafter
2113   {#3}{#1#2}%
2114 }%

#1 = B, #2 = |A|

2115 \def\xint@pow@Anonneg@ #1#2%
2116 {%
2117   \ifcase\xint@Cmp {#2}{1}%
2118   \expandafter\xint@pow@AisOne
2119   \or
2120   \expandafter\xint@pow@AatleastTwo
2121   \else
2122   \expandafter\xint@pow@AisZero
2123   \fi
2124   {#1}{#2}%
2125 }%
2126 \def\xint@pow@AisOne #1#2{ 1}%

#1 = B

2127 \def\xint@pow@AisZero #1#2%
2128 {%
2129   \ifcase\xint@Sgn {#1}%
2130   \xint@afterfi { 1}%
2131   \or
2132   \xint@afterfi { 0}%
2133   \else
2134   \xint@afterfi {\xintError:DivisionByZero\space 0}%
2135   \fi
2136 }%
2137 \def\xint@pow@AatleastTwo #1%
2138 {%

```

```

2139   \ifcase\XINT@Sgn {#1}
2140     \expandafter\XINT@pow@BisZero
2141   \or
2142     \expandafter\XINT@pow@checkLength
2143   \else
2144     \expandafter\XINT@pow@BisNegative
2145   \fi
2146   {#1}%
2147 }%
2148 \def\XINT@pow@BisNegative #1#2{\xintError:FractionRoundedToZero\space 0}%
2149 \def\XINT@pow@BisZero #1#2{ 1}%

B = #1 > 0, A = #2 > 1

2150 \def\XINT@pow@checkLength #1#2%
2151 {%
2152   \ifnum\xintLen{#1} >9
2153     \expandafter\XINT@pow@BtooBig
2154   \else
2155     \expandafter\XINT@pow@loop
2156   \fi
2157   {#1}{#2}\XINT@posprod
2158   \xint@UNDEF
2159   \xint@undef\xint@undef\xint@undef\xint@undef
2160   \xint@undef\xint@undef\xint@undef\xint@undef
2161   \xint@UNDEF
2162 }%
2163 \def\XINT@pow@BtooBig #1\xint@UNDEF #2\xint@UNDEF
2164           {\xintError:ExponentTooBig\space 0}%
2165 \def\XINT@pow@loop #1#2%
2166 {%
2167   \ifnum #1 = 1
2168     \expandafter\XINT@pow@loop@end
2169   \else
2170     \xint@afterfi{\expandafter\XINT@pow@loop@a
2171       \expandafter{\the\numexpr 2*(#1/2)-#1\expandafter }%
2172       % b mod 2
2173       \expandafter{\the\numexpr #1-#1/2\expandafter }%
2174       % [b/2]
2175       \expandafter{\romannumeral0\xint sqr{#2}} }%
2176   \fi
2177   {{#2}}%
2178 }%
2179 \def\XINT@pow@loop@end {\romannumeral0\XINT@rord@main {} \relax }%
2180 \def\XINT@pow@loop@a #1%
2181 {%
2182   \ifnum #1 = 1
2183     \expandafter\XINT@pow@loop
2184   \else
2185     \expandafter\XINT@pow@loop@throwaway
2186   \fi
2187 }%
2188 \def\XINT@pow@loop@throwaway #1#2#3%

```

```

2189 {%
2190   \XINT@pow@loop {#1}{#2}%
2191 }%

```

12.25 \xintDivision, \xintQuo, \xintRem

```

2192 \def\xintQuo {\romannumeral0\xintquo }%
2193 \def\xintRem {\romannumeral0\xintrem }%
2194 \def\xintquo {\expandafter
2195           \xint@firstoftwo@andstop
2196           \romannumeral0\xintdivision }%
2197 \def\xintrem {\expandafter
2198           \xint@secondoftwo@andstop
2199           \romannumeral0\xintdivision }%
2200 #1 = A, #2 = B. On calcule le quotient de A par B

2200 \def\xintDivision {\romannumeral0\xintdivision }%
2201 \def\xintdivision #1%
2202 {%
2203   \expandafter\expandafter\expandafter
2204   \xint@division
2205   \expandafter\expandafter\expandafter
2206   {#1}%
2207 }%
2208 \def\xint@division #1#2%
2209 {%
2210   \expandafter\expandafter\expandafter
2211   \XINT@div@fork #2\Z #1\Z
2212 }%
2213 \def\XINT@Division #1#2{\romannumeral0\XINT@div@fork #2\Z #1\Z }%

#1#2 = 2e input = diviseur = B
#3#4 = 1er input = divisé = A

2214 \def\XINT@div@fork #1#2\Z #3#4\Z
2215 {%
2216   \xint@UDzerofork
2217   #1\dummy \XINT@div@BisZero
2218   #3\dummy \XINT@div@AisZero
2219   0\dummy
2220   {\xint@UDsignfork
2221     #1\dummy \XINT@div@BisNegative % B < 0
2222     #3\dummy \XINT@div@AisNegative % A < 0, B > 0
2223     -\dummy \XINT@div@plusplus    % B > 0, A > 0
2224   \xint@UDkrof }%
2225   \xint@UDkrof
2226   {#2}{#4}#1#3% #1#2=B, #3#4=A
2227 }%
2228 \def\XINT@div@BisZero #1#2#3#4%
2229   {\xintError:DivisionByZero\space {0}{0}}%
2230 \def\XINT@div@AisZero #1#2#3#4{ {0}{0}}%

```

jusqu'à présent c'est facile.

```

minusplus signifie  $B < 0$ ,  $A > 0$ 
plusminus signifie  $B > 0$ ,  $A < 0$ 
Ici #3#1 correspond au diviseur  $B$  et #4#2 au divisé  $A$ 

2231 \def\xint@div@plusplus #1#2#3#4%
2232 {%
2233   \xint@div@prepare {#3#1}{#4#2}%
2234 }%

B = #3#1 < 0, A non nul positif ou négatif

2235 \def\xint@div@BisNegative #1#2#3#4%
2236 {%
2237   \expandafter\xint@div@BisNegative@post
2238   \romannumeral0\xint@div@fork #1\Z #4#2\Z
2239 }%
2240 \def\xint@div@BisNegative@post #1#2%
2241 {%
2242   \expandafter\space\expandafter
2243   {\romannumeral0\xint@opp #1}{#2}%
2244 }%

B = #3#1 > 0, A =-#2< 0

2245 \def\xint@div@AisNegative #1#2#3#4%
2246 {%
2247   \expandafter\xint@div@AisNegative@post
2248   \romannumeral0\xint@div@prepare {#3#1}{#2}{#3#1}%
2249 }%
2250 \def\xint@div@AisNegative@post #1#2%
2251 {%
2252   \ifcase\xintSgn {#2}
2253     \expandafter \xint@div@AisNegative@zerorem
2254   \or
2255     \expandafter \xint@div@AisNegative@posrem
2256   \fi
2257 {#1}{#2}%
2258 }%

en #3 on a une copie de  $B$  (à l'endroit)

2259 \def\xint@div@AisNegative@zerorem #1#2#3%
2260 {%
2261   \expandafter\space\expandafter
2262   {\romannumeral0\xint@opp #1}{0}%
2263 }%

#1 = quotient, #2 = reste, #3 = diviseur initial (à l'endroit)

2264 \def\xint@div@AisNegative@posrem #1%
2265 {%
2266   \expandafter
2267   \xint@div@AisNegative@posrem@b
2268   \expandafter
2269   {\romannumeral0\xintopp {\xint@Add{#1}{1}}}%
2270 }%

```

```

remplace Reste par B - Reste, après avoir remplacé Q par -(Q+1)
de sorte que la formule a = qb + r, 0<= r < |b| est valable

2271 \def\XINT@div@AisNegative@posrem@b #1#2#3%
2272 {%
2273     \expandafter
2274         \xint@exchangetwo@keepbraces@andstop
2275     \expandafter
2276     {\romannumeral0\XINT@sub {#3}{#2}}{#1}%
2277 }%

par la suite A et B sont > 0.
#1 = B. Pour le moment à l'endroit.

2278 \def\XINT@div@prepare #1%
2279 {%
2280     \expandafter
2281         \XINT@div@prepareB@a
2282     \expandafter
2283     {\romannumeral0\XINT@length {#1}}{#1}% B > 0 ici
2284 }%

Calcul du plus petit K = 4n >= longueur de B

2285 \def\XINT@div@prepareB@a #1%
2286 {%
2287     \expandafter\XINT@div@prepareB@b\expandafter
2288     {\the\numexpr 4*((#1+1)/4)\relax}{#1}%
2289 }%

#1 = K

2290 \def\XINT@div@prepareB@b #1#2%
2291 {%
2292     \expandafter\XINT@div@prepareB@c \expandafter
2293     {\the\numexpr #1-#2\relax}{#1}%
2294 }%

#1 = c

2295 \def\XINT@div@prepareB@c #1%
2296 {%
2297     \ifcase #1
2298         \expandafter\XINT@div@prepareB@di
2299     \or \expandafter\XINT@div@prepareB@dii
2300     \or \expandafter\XINT@div@prepareB@diii
2301     \else \expandafter\XINT@div@prepareB@div
2302     \fi
2303 }%
2304 \def\XINT@div@prepareB@di {\XINT@div@prepareB@e {}{0}}%
2305 \def\XINT@div@prepareB@dii {\XINT@div@prepareB@e {0}{1}}%
2306 \def\XINT@div@prepareB@diii {\XINT@div@prepareB@e {00}{2}}%
2307 \def\XINT@div@prepareB@div {\XINT@div@prepareB@e {000}{3}}%

```

```

#1 = zéros à rajouter à B, #2=c, #3=K, #4 = B

2308 \def\xint@div@prepareB@e #1#2#3#4%
2309 {%
2310     \xint@div@prepareB@f #4#1\Z {#3}{#2}{#1}%
2311 }%

x = #1#2#3#4 = 4 premiers chiffres de B. #1 est non nul.
Ensuite on renverse B pour calculs plus rapides par la suite.

2312 \def\xint@div@prepareB@f #1#2#3#4#5\Z
2313 {%
2314     \expandafter
2315     \xint@div@prepareB@g
2316     \expandafter
2317     {\romannumeral0\xint@rev {#1#2#3#4#5}}{#1#2#3#4}%
2318 }%

#3= K, #4 = c, #5= {} ou {0} ou {00} ou {000}, #6 = A initial
#1 = B préparé et renversé, #2 = x = quatre premiers chiffres
On multiplie aussi A par  $10^c$ .
B, x, K, c, {} ou {0} ou {00} ou {000}, A initial

2319 \def\xint@div@prepareB@g #1#2#3#4#5#6%
2320 {%
2321     \xint@div@prepareA@a {#6#5}{#2}{#3}{#1}{#4}%
2322 }%

A, x, K, B, c,

2323 \def\xint@div@prepareA@a #1%
2324 {%
2325     \expandafter
2326     \xint@div@prepareA@b
2327     \expandafter
2328     {\romannumeral0\xint@length {#1}}{#1}%
2329 }%

L0, A, x, K, B, ...

2330 \def\xint@div@prepareA@b #1%
2331 {%
2332     \expandafter\xint@div@prepareA@c\expandafter
2333     {\the\numexpr 4*((#1+1)/4)\relax}{#1}%
2334 }%

L, L0, A, x, K, B, ...

```

12 Package **xint** implementation

```

2335 \def\XINT@div@prepareA@c #1#2%
2336 {%
2337     \expandafter\XINT@div@prepareA@d \expandafter
2338         {\the\numexpr #1-#2\relax}{#1}%
2339 }%
2340 \def\XINT@div@prepareA@d #1%
2341 {%
2342     \ifcase #1
2343         \expandafter\XINT@div@prepareA@di
2344     \or \expandafter\XINT@div@prepareA@dii
2345     \or \expandafter\XINT@div@prepareA@diii
2346     \else \expandafter\XINT@div@prepareA@div
2347     \fi
2348 }%
2349 \def\XINT@div@prepareA@di {\XINT@div@prepareA@e {}}%
2350 \def\XINT@div@prepareA@dii {\XINT@div@prepareA@e {0}}%
2351 \def\XINT@div@prepareA@diii {\XINT@div@prepareA@e {00}}%
2352 \def\XINT@div@prepareA@div {\XINT@div@prepareA@e {000}}%

#1#3 = A préparé, #2 = longueur de ce A préparé,
2353 \def\XINT@div@prepareA@e #1#2#3%
2354 {%
2355     \XINT@div@startswitch {#1#3}{#2}%
2356 }%

A, L, x, K, B, c

2357 \def\XINT@div@startswitch #1#2#3#4%
2358 {%
2359     \ifnum #2 > #4
2360         \expandafter\XINT@div@body@a
2361     \else
2362         \ifnum #2 = #4
2363             \expandafter\expandafter\expandafter
2364                 \XINT@div@final@a
2365         \else
2366             \expandafter\expandafter\expandafter
2367                 \XINT@div@finished@a
2368         \fi\fi {#1}{#4}{#3}{0000}{#2}%
2369 }%

A, K, x, Q, L, B, c
---- "Finished"

2370 \def\XINT@div@finished@a #1#2#3%
2371 {%
2372     \expandafter
2373         \XINT@div@finished@b
2374     \expandafter
2375         {\romannumeral0\XINT@cu{#1}}%
2376 }%

A, Q, L, B, c
no leading zeros in A at this stage

```

12 Package **xint** implementation

```

2377 \def\XINT@div@finished@b #1#2#3#4#5%
2378 {%
2379     \ifcase \XINT@Sgn {#1}%
2380         \xint@afterfi {\XINT@div@finished@c {0}}%
2381     \or
2382         \xint@afterfi {\expandafter\XINT@div@finished@c
2383                         \expandafter
2384                         {\romannumeral0\XINT@dsh@checksignx #5\Z {#1}}}}%
2385     \fi
2386 {#2}%
2387 }%


Reste Final, Q à renverser
#2 = Quotient, #1 = Reste.

2388 \def\XINT@div@finished@c #1#2%
2389 {%
2390     \expandafter
2391         \space
2392     \expandafter
2393         {\romannumeral0\expandafter\xint@cleanupzeros@andstop
2394             \romannumeral0\XINT@rev {#2}}{#1}%
2395 }%


---- "Final"
A, K, x, Q, L, B, c

2396 \def\XINT@div@final@a #1%
2397 {%
2398     \XINT@div@final@b #1\Z
2399 }%
2400 \def\XINT@div@final@b #1#2#3#4#5\Z
2401 {%
2402     \xint@quatrezeros #1#2#3#4\xint@div@final@c0000%
2403     \XINT@div@final@c {#1#2#3#4}{#1#2#3#4#5}%
2404 }%
2405 \def\xint@div@final@c0000\XINT@div@final@c #1%
2406                         {\XINT@div@finished@a }%


a, A, K, x, Q, L, B ,c

2407 \def\XINT@div@final@c #1#2#3#4%
2408 {%
2409     \expandafter
2410     \XINT@div@final@d
2411     \expandafter
2412     {\the\numexpr #1/#4\relax}{#2}%
2413 }%


q, A, Q, L, B à l'envers sur 4n, c
1.01 code ré-écrit pour optimisations diverses

```

```

2414 \def\xint@div@final@d #1#2#3#4#5% q,A,Q,L,B puis c
2415 {%
2416     \expandafter
2417         \xint@div@final@da
2418     \expandafter
2419         {\romannumeral0\xint@mul@M {#1}#5\Z\Z\Z\Z }%
2420         {\romannumeral0\xint@cleanupzeros@andstop #2}%
2421         {#1}{#3}{#5}%
2422 }%
2423 \def\xint@div@final@da #1#2%
2424 {%
2425     \expandafter\xint@div@final@db\expandafter {#2}{#1}%
2426 }%
2427 \def\xint@div@final@db #1#2% A,qB, puis q,Q,B,c
2428 {%
2429     \ifcase\xint@Geq {#1}{#2}
2430         \expandafter\xint@div@final@dc % A < qB
2431         \or\expandafter\xint@div@final@e % A au moins qB
2432         \fi
2433         {#1}{#2}%
2434 }%
2435 \def\xint@div@final@e #1#2#3#4#5% A,qB,q,Q,B,puis c
2436 {%
2437     \expandafter\xint@div@final@f
2438     \expandafter{\romannumeral0\xintsub {#1}{#2}}%
2439     {\romannumeral0\xintadd {\xint@Rev@andcleanupzeros{#4}}{#3}}%
2440 }%
2441 \def\xint@div@final@dc #1#2#3% A sans leading zeros,trash,q,Q,B,c
2442 {%
2443     \expandafter\xint@div@final@dd
2444     \expandafter{\the\numexpr #3-1\relax}{#1}%
2445 }%
2446 \def\xint@div@final@dd #1#2#3#4% q,A,Q,B puis c
2447 {%
2448     \expandafter\xint@div@final@f
2449     \expandafter{\romannumeral0\xintsub
2450                 {#2}{\romannumeral0\xint@mul@M {#1}#4\Z\Z\Z\Z } }%
2451     {\romannumeral0\xintadd {\xint@Rev@andcleanupzeros{#3}}{#1}}%
2452 }%
2453 \def\xint@div@final@f #1#2#3% R,Q à développer,c
2454 {%
2455     \ifcase \xint@Sgn {#1}
2456         \xint@afterfi {\xint@div@final@end {0}}%
2457     \or
2458         \xint@afterfi {\expandafter\xint@div@final@end
2459                         \expandafter % pas de leading zeros dans #1=R
2460                         {\romannumeral0\xint@dsh@checksignx #3\Z {#1}} }%
2461     \fi
2462     {#2}%
2463 }%
2464 \def\xint@div@final@end #1#2%
2465 {%
2466     \expandafter\space\expandafter {#2}{#1}%

```

```

2467 }%
Boucle Principale
A, K, x, Q, L, B, c

2468 \def\xint@div@body@a #1%
2469 {%
2470     \xint@div@body@b #1\Z
2471 }%
2472 \def\xint@div@body@b #1#2#3#4#5#6#7#8#9\Z
2473 {%
2474     \xint@div@body@c
2475     {#1#2#3#4#5#6#7#8#9}%
2476     {#1#2#3#4#5#6#7#8}%
2477 }%
A, a, K, x, Q, L, B, c

2478 \def\xint@div@body@c #1#2#3%
2479 {%
2480     \xint@div@body@d {#3}{}}#1\Z {#2}{#3}%
2481 }%
2482 \def\xint@div@body@d #1#2#3#4#5#6%
2483 {%
2484     \ifnum #1 > 0
2485         \expandafter
2486         \xint@div@body@d
2487         \expandafter
2488         {\the\numexpr #1-4\expandafter }%
2489     \else
2490         \expandafter
2491         \xint@div@body@e
2492     \fi
2493     {#6#5#4#3#2}%
2494 }%
2495 \def\xint@div@body@e #1#2\Z #3%
2496 {%
2497     \xint@div@body@f {#3}{#1}{#2}%
2498 }%
a, alpha, alpha', K, x, Q, L, B, c

2499 \def\xint@div@body@f #1#2#3#4#5#6#7#8%
2500 {%
2501     \expandafter\xint@div@body@g
2502     \expandafter
2503     {\the\numexpr (#1+(#5+1)/2)/(#5+1)-1\relax }%
2504     {#2}{#8}{#4}{#5}{#3}{#6}{#7}{#8}%
2505 }%
q1, alpha, B, K, x, alpha', Q, L, B, c

```

```

2506 \def\XINT@div@body@g #1#2#3%
2507 {%
2508     \expandafter
2509         \XINT@div@body@h
2510     \romannumeral0\XINT@div@sub@xp{%
2511         {\romannumeral0\XINT@prod@mul@enter #3\W\X\Y\Z #1\W\X\Y\Z }%
2512         {#2}\Z
2513         {#3}{#1}%
2514 }%
2515 alpha1 = alpha-q1 B, \Z, B, q1, K, x, alpha', Q, L, B, c
2516 \def\XINT@div@body@h #1#2#3#4#5#6#7#8#9\Z
2517 {%
2518     \ifnum #1#2#3#4>0
2519         \xint@afterfi{\XINT@div@body@i {#1#2#3#4#5#6#7#8}}%
2520     \else
2521         \expandafter\XINT@div@body@k
2522     \fi
2523 {#1#2#3#4#5#6#7#8#9}%
2524 }%
2525 a1, alpha1, B, q1, K, x, alpha', Q, L, B, c
2526 \def\XINT@div@body@i #1#2#3#4#5#6%
2527 {%
2528     \expandafter\XINT@div@body@j
2529     \expandafter{\the\numexpr (#1+(#6+1)/2)/(#6+1)-1\relax }%
2530     {#2}{#3}{#4}{#5}{#6}%
2531 }%
2532 q2, alpha1, B, q1, K, x, alpha', Q, L, B, c
2533 \def\XINT@div@body@j #1#2#3#4%
2534 {%
2535     \expandafter
2536         \XINT@div@body@l
2537     \expandafter{\romannumeral0\XINT@div@sub@xp{%
2538         {\romannumeral0\XINT@prod@mul@enter #3\W\X\Y\Z #1\W\X\Y\Z }%
2539         {\XINT@Rev{#2}}} }%
2540     {#4+#1}%
2541 }%
2542 alpha2, q1+q2, K, x, alpha', Q, L, B, c
2543 attention body@j -> body@l
2544 alpha1, B, q=q1, K, x, alpha', Q, L, B, c
2545 \def\XINT@div@body@k #1#2%
2546 {%
2547     \XINT@div@body@l {#1}%
2548 }%
2549 alpha2, q= q1+q2, K, x, alpha', Q, L, B, c

```

```

2543 \def\xint@div@body@l #1#2#3#4#5#6#7%
2544 {%
2545     \expandafter
2546         \xint@div@body@m
2547     \the\numexpr 100000000+#2\relax
2548     {#6}{#3}{#7}{#1#5}{#4}%
2549 }%

    chiffres de q, Q, K, L, A', x, B, c

2550 \def\xint@div@body@m #1#2#3#4#5#6#7#8#9%
2551 {%
2552     \ifnum #2#3#4#5>0
2553         \xint@afterfi {\xint@div@body@n {#9#8#7#6#5#4#3#2}}%
2554     \else
2555         \xint@afterfi {\xint@div@body@n {#9#8#7#6}}%
2556     \fi
2557 }%

    q renversé, Q, K, L, A', x, B, c

2558 \def\xint@div@body@n #1#2%
2559 {%
2560     \expandafter\xint@div@body@o\expandafter
2561     {\romannumeral0\xint@sum@A 0{}#1\W\X\Y\Z #2\W\X\Y\Z }%
2562 }%

    q+Q, K, L, A', x, B, c

2563 \def\xint@div@body@o #1#2#3#4%
2564 {%
2565     \xint@div@body@p {#3}{#2}{ }#4\Z {#1}%
2566 }%

    L, K, {}, A'\Z, q+Q, x, B, c

2567 \def\xint@div@body@p #1#2#3#4#5#6#7%
2568 {%
2569     \ifnum #1 > #2
2570         \xint@afterfi
2571         {\ifnum #4#5#6#7 > 0
2572             \expandafter\xint@div@body@q
2573         \else
2574             \expandafter\xint@div@body@repeatp
2575         \fi }%
2576     \else
2577         \expandafter\xint@div@gotofinal@a
2578     \fi
2579     {#1}{#2}{#3}#4#5#6#7%
2580 }%

    L, K, zeros, A' avec moins de zéros\Z, q+Q, x, B, c

```

12 Package **xint** implementation

```

2581 \def\xint@div@body@repeatp #1#2#3#4#5#6#7%
2582 {%
2583     \expandafter
2584     \xint@div@body@p
2585     \expandafter
2586     {\the\numexpr #1-4\relax}{#2}{0000#3}%
2587 }%
L -> L-4, zeros->zeros+0000, répéter jusqu'à ce que soit L=K
soit on ne trouve plus 0000
nouveau L, K, zeros, nouveau A=#4, Q+q, x, B, c

2588 \def\xint@div@body@q #1#2#3#4\Z #5#6%
2589 {%
2590     \xint@div@body@a {#4}{#2}{#6}{#3#5}{#1}%
2591 }%
A, K, x, Q, L, B, c --> iterate
-----
Boucle Principale achevée
ATTENTION IL FAUT AJOUTER 4 ZEROS DE MOINS QUE CEUX
QUI ONT ÉTÉ PRÉPARÉS DANS #3!!
L, K (L=K), zeros, A\Z, Q, x, B, c

2592 \def\xint@div@gotofinal@a #1#2#3#4\Z %
2593 {%
2594     \xint@div@gotofinal@b #3\Z {#4}{#1}%
2595 }%
zeros\Z, A, L=K, Q, x, B, c

2596 \def\xint@div@gotofinal@b 0000#1\Z #2#3#4#5%
2597 {%
2598     \xint@div@final@a {#2}{#3}{#5}{#1#4}{#3}%
2599 }%
A, L=K, x, Q avec zéros, L, B, c
La soustraction spéciale. Étendre deux fois les arguments
pour \xint@div@sub@enter longueur multiple de 4 on sait que #2>#1,

2600 \def\xint@div@sub@xp@xp #1%
2601 {%
2602     \expandafter
2603     \xint@div@sub@xp@xp@
2604     \expandafter
2605     {#1}%
2606 }%
2607 \def\xint@div@sub@xp@xp@ #1#2%
2608 {%
2609     \expandafter\expandafter\expandafter
2610     \xint@div@sub@xp@xp@@
2611     #2\W\X\Y\Z #1\W\X\Y\Z
2612 }%
2613 \def\xint@div@sub@xp@xp@@
2614 {%

```

```

2615      \XINT@div@sub@a 1{ }%
2616 }%
2617 \def\XINT@div@sub@a #1#2#3#4#5#6%
2618 {%
2619     \xint@w
2620     #3\xint@div@sub@az
2621     \W\XINT@div@sub@b #1{#3#4#5#6}{#2}%
2622 }%
2623 \def\XINT@div@sub@b #1#2#3#4\W\X\Y\Z #5#6#7#8%
2624 {%
2625     \xint@w
2626     #5\xint@div@sub@bz
2627     \W\XINT@div@sub@onestep #1#2{#8#7#6#5}{#3}{#4}\W\X\Y\Z
2628 }%
2629 \def\XINT@div@sub@onestep #1#2#3#4#5#6%
2630 {\expandafter
2631     \XINT@div@sub@backtoa\the\numexpr 11#5#4#3#2-#6+#1-1\relax.%
2632 }%
2633 \def\XINT@div@sub@backtoa #1#2#3.#4%
2634 {%
2635     \XINT@div@sub@a #2{#3#4}%
2636 }%
2637 \def\xint@div@sub@bz
2638     \W\XINT@div@sub@onestep #1#2#3#4#5#6#7%
2639 {%
2640     \xint@UDzerofork
2641         #1\dummy \XINT@div@sub@c %
2642         0\dummy \XINT@div@sub@d % pas de retenue
2643     \xint@UDkrof
2644     {#7}#2#3#4#5%
2645 }%
2646 \def\XINT@div@sub@d #1#2\W\X\Y\Z
2647 {%
2648     \expandafter\space
2649     \romannumeral0%
2650     \XINT@rord@main {}#2%
2651     \xint@UNDEF
2652         \xint@undef\xint@undef\xint@undef\xint@undef
2653         \xint@undef\xint@undef\xint@undef\xint@undef
2654         \xint@UNDEF
2655     #1%
2656 }%
2657 \def\XINT@div@sub@c #1#2#3#4#5%
2658 {%
2659     \xint@w
2660     #2\xint@div@sub@cz
2661     \W\XINT@div@sub@ac@onestep {#5#4#3#2}{#1}%
2662 }%
2663 \def\XINT@div@sub@ac@onestep #1%
2664 {\expandafter
2665     \XINT@div@sub@backtoc\the\numexpr 11#1-1\relax.%
2666 }%

```

```

2667 \def\xint@div@sub@backtoC #1#2#3.#4%
2668 {%
2669   \XINT@div@sub@AC@checkcarry #2{#3#4}% la retenue va \^etre examin\'ee
2670 }%
2671 \def\xint@div@sub@AC@checkcarry #1%
2672 {%
2673   \xint@one #1\xint@div@sub@AC@nocarry 1\XINT@div@sub@C
2674 }%
2675 \def\xint@div@sub@AC@nocarry 1\XINT@div@sub@C #1#2\W\X\Y\Z
2676 {%
2677   \expandafter\space
2678   \romannumeral0%
2679   \XINT@rord@main {}#2%
2680   \xint@UNDEF
2681   \xint@undef\xint@undef\xint@undef\xint@undef
2682   \xint@undef\xint@undef\xint@undef\xint@undef
2683   \xint@UNDEF
2684   #1%
2685 }%
2686 \def\xint@div@sub@cz\W\XINT@div@sub@AC@onestep #1#2{ #2}%
2687 \def\xint@div@sub@az\W\XINT@div@sub@B #1#2#3#4\Z { #3}%
-----
```

DECIMAL OPERATIONS: FIRST DIGIT, LASTDIGIT, ODDNESS,
 MULTIPLICATION BY TEN, QUOTIENT BY TEN, QUOTIENT OR
 MULTIPLICATION BY POWER OF TEN, SPLIT OPERATION.

12.26 \xintFDg

FIRST DIGIT

```

2688 \def\xintFDg {\romannumeral0\xintfdg }%
2689 \def\xintfdg #1%
2690 {%
2691   \expandafter\expandafter\expandafter
2692   \XINT@fdg #1\W\Z
2693 }%
2694 \def\xint@FDg #1{\romannumeral0\XINT@fdg #1\W\Z }%
2695 \def\xint@fdg #1#2%
2696 {%
2697   \xint@xp@andstop
2698   \xint@UDzerominusfork
2699   #1-\dummy {\expandafter 0}% zero
2700   0#1\dummy {\expandafter #2}% negative
2701   0-\dummy {\expandafter #1}% positive
2702   \xint@UDkrof
2703   \xint@z
2704 }%
```

12.27 \xintLDg

LAST DIGIT

```

2705 \def\xintLDg {\romannumeral0\xintldg }%
2706 \def\xintldg #1%
2707 {%
2708     \expandafter\expandafter\expandafter
2709         \XINT@ldg
2710     \expandafter\expandafter\expandafter
2711         {#1}%
2712 }%
2713 \def\XINT@LDg #1{\romannumeral0\XINT@ldg {#1}}%
2714 \def\XINT@ldg #1%
2715 {%
2716     \expandafter
2717     \XINT@ldg@
2718     \romannumeral0\XINT@rev {#1}\Z
2719 }%
2720 \def\XINT@ldg@ #1%
2721 {%
2722     \expandafter\space\expandafter #1\xint@z
2723 }%

```

12.28 \xintOdd

ODDNESS

```

2724 \def\xintOdd {\romannumeral0\xintodd }%
2725 \def\xintodd #1%
2726 {%
2727     \ifodd\xintLDg{#1}
2728         \xint@afterfi{ 1}%
2729     \else
2730         \xint@afterfi{ 0}%
2731     \fi
2732 }%
2733 \def\XINT@Odd #1%
2734 {\romannumeral0%
2735     \ifodd\XINT@LDg{#1}
2736         \xint@afterfi{ 1}%
2737     \else
2738         \xint@afterfi{ 0}%
2739     \fi
2740 }%

```

12.29 \xintDSL

DECIMAL SHIFT LEFT (=MULTIPLICATION PAR 10)

```

2741 \def\xintDSL {\romannumeral0\xintdsl }%
2742 \def\xintdsl #1%
2743 {%
2744     \expandafter\expandafter\expandafter
2745         \XINT@dsl #1\Z
2746 }%
2747 \def\XINT@DSL #1{\romannumeral0\XINT@dsl #1\Z }%

```

```

2748 \def\XINT@dsl #1%
2749 {%
2750     \xint@zero #1\xint@dsl@zero 0\XINT@dsl@ #1%
2751 }%
2752 \def\xint@dsl@zero 0\XINT@dsl@ 0#1\Z { 0}%
2753 \def\XINT@dsl@ #1\Z { #10}%

```

12.30 \xintDSR

DECIMAL SHIFT RIGHT (=DIVISION PAR 10)

```

2754 \def\xintDSR {\romannumeral0\xintdsr }%
2755 \def\xintdsr #1%
2756 {%
2757     \expandafter\expandafter\expandafter
2758         \XINT@dsr@a
2759     \expandafter\expandafter\expandafter
2760         {#1}\W\Z
2761 }%
2762 \def\XINT@DSR #1{\romannumeral0\XINT@dsr@a {#1}\W\Z }%
2763 \def\XINT@dsr@a
2764 {%
2765     \expandafter
2766         \XINT@dsr@b
2767     \romannumeral0\XINT@rev
2768 }%
2769 \def\XINT@dsr@b #1#2#3\Z
2770 {%
2771     \xint@w #2\xint@dsr@onedigit\W
2772     \xint@minus #2\xint@dsr@onedigit-%
2773     \expandafter
2774         \XINT@dsr@removew
2775     \romannumeral0\XINT@rev {#2#3}%
2776 }%
2777 \def\xint@dsr@onedigit #1\XINT@rev #2{ 0}%
2778 \def\XINT@dsr@removew #1\W { }%

```

12.31 \xintDSH, \xintDSHr

```

DECIMAL SHIFTS
\xintDSH {x}{A}
si x <= 0, fait A -> A.10^(|x|)
si x > 0, et A >=0, fait A -> quo(A,10^(x))
si x > 0, et A < 0, fait A -> -quo(-A,10^(x))
(donc pour x > 0 c'est comme DSR itéré x fois)
\xintDSHr donne le 'reste'.

2779 \def\xintDSHr {\romannumeral0\xintdshr }%
2780 \def\xintdshr #1%
2781 {\expandafter\expandafter\expandafter
2782             \XINT@dsh@checkxpositive #1\Z
2783 }%
2784 \def\XINT@dsh@checkxpositive #1%

```

```

2785 {%
2786     \xint@UDzerominusfork
2787     0#1\dummy \XINT@dsh@xzeroorneg
2788     #1-\dummy \XINT@dsh@xzeroorneg
2789     0-\dummy \XINT@dsh@xpositive
2790     \xint@UDkrof #1%
2791 }%
2792 \def\xINT@dsh@xzeroorneg #1\Z #2{ 0}%
2793 \def\xINT@dsh@xpositive #1\Z
2794 {%
2795     \expandafter
2796     \xint@secondoftwo@andstop
2797     \romannumeral0\xintdsx {#1}%
2798 }%
2799 \def\xintDSH {\romannumeral0\xintdsh }%
2800 \def\xintdsh #1#2%
2801 {%
2802     \expandafter\expandafter\expandafter
2803         \xint@dsh
2804     \expandafter\expandafter\expandafter
2805         {#2}{#1}%
2806 }%
2807 \def\xint@dsh #1#2%
2808 {%
2809     \expandafter\expandafter\expandafter
2810         \XINT@dsh@checksignx
2811     #2\Z {#1}%
2812 }%
2813 \def\xINT@dsh@checksignx #1%
2814 {%
2815     \xint@UDzerominusfork
2816     #1-\dummy \XINT@dsh@xiszero
2817     0#1\dummy \XINT@dsx@xisNeg
2818     0-\dummy {\XINT@dsh@xisPos #1}%
2819     \xint@UDkrof
2820 }%
2821 \def\xINT@dsh@xiszero #1\Z #2{ #2}%
2822 \def\xINT@dsh@xisPos #1\Z #2%
2823 {%
2824     \expandafter
2825     \xint@firstoftwo@andstop
2826     \romannumeral0\XINT@dsx@checksignA #2\Z {#1}%
2827 }%

```

12.32 \xintDSx

Je fais cette routine pour la version 1.01, après modification de `\xintDecSplit`. Dorénavant `\xintDSx` fera appel à `\xintDecSplit` et de même `\xintDSH` fera appel à `\xintDSx`. J'ai donc supprimé entièrement l'ancien code de `\xintDSH` et re-écrit entièrement celui de `\xintDecSplit` pour x positif.

```

si x <= 0, fait A -> A.10^(|x|)
si x > 0, et A >=0, fait A -> {quo(A,10^(x))}{rem(A,10^(x))}

```

12 Package **xint** implementation

si $x > 0$, et $A < 0$, d'abord on calcule $\{ \text{quo}(-A, 10^x) \} \{ \text{rem}(-A, 10^x) \}$
puis, si le premier n'est pas nul on lui donne le signe -
si le premier est nul on donne le signe - au second.
On peut donc toujours reconstituer l'original A par $10^x Q \pm R$
où il faut prendre le signe plus si Q est positif ou nul et le signe moins si
 Q est strictement négatif.

```

2828 \def\xintDSx {\romannumeral0\xintdsx }%
2829 \def\xintdsx #1#2%
2830 {%
2831     \expandafter\expandafter\expandafter
2832         \xint@dsx
2833     \expandafter\expandafter\expandafter
2834         {#2}{#1}%
2835 }%
2836 \def\xint@dsx #1#2%
2837 {%
2838     \expandafter\expandafter\expandafter
2839         \XINT@dsx@checksignx
2840     #2\Z {#1}%
2841 }%
2842 \def\XINT@DSx #1#2{\romannumeral0\XINT@dsx@checksignx #1\Z {#2}}%
2843 \def\XINT@dsx #1#2{\XINT@dsx@checksignx #1\Z {#2}}%
2844 \def\XINT@dsx@checksignx #1%
2845 {%
2846     \xint@UDzerominusfork
2847     #1-\dummy \XINT@dsx@xisZero
2848     0#1\dummy \XINT@dsx@xisNeg
2849     0-\dummy {\XINT@dsx@xisPos #1}%
2850     \xint@UDkrof
2851 }%
2852 \def\XINT@dsx@xisZero #1\Z #2{ {#2}{0}}%
2853 \def\XINT@dsx@xisNeg #1\Z
2854 {%
2855     \ifnum\XINT@Len {#1} > 9
2856         \xint@afterfi {\xintError:TooBigDecimalShift
2857                         \XINT@dsx@toobigx }%
2858     \else
2859         \expandafter \XINT@dsx@zeroloop
2860     \fi
2861     {#1}\Z
2862 }%
2863 \def\XINT@dsx@toobigx #1\Z #2{ {#2}}%
2864 \def\XINT@dsx@zeroloop #1%
2865 {%
2866     \ifcase #1
2867         \expandafter \XINT@dsx@exit
2868     \or
2869         \expandafter \XINT@dsx@exiti
2870     \or
2871         \expandafter \XINT@dsx@exitii
2872     \or
2873         \expandafter \XINT@dsx@exitiii

```

```

2874      \or
2875          \expandafter \XINT@dsx@exitiv
2876      \or
2877          \expandafter \XINT@dsx@exitv
2878      \or
2879          \expandafter \XINT@dsx@exitvi
2880      \or
2881          \expandafter \XINT@dsx@exitvii
2882  \else
2883      \xint@afterfi
2884      {\expandafter
2885          \XINT@dsx@zeroloop
2886          \expandafter {\the\numexpr #1-8}00000000%
2887      }%
2888  \fi
2889 }%
2890 \def\XINT@dsx@exit #1\Z
2891             {\XINT@dsx@addzeros {#1}}%
2892 \def\XINT@dsx@exiti #1\Z
2893             {\XINT@dsx@addzeros {0#1}}%
2894 \def\XINT@dsx@exitii #1\Z
2895             {\XINT@dsx@addzeros {00#1}}%
2896 \def\XINT@dsx@exitiii #1\Z
2897             {\XINT@dsx@addzeros {000#1}}%
2898 \def\XINT@dsx@exitiv #1\Z
2899             {\XINT@dsx@addzeros {0000#1}}%
2900 \def\XINT@dsx@exitv #1\Z
2901             {\XINT@dsx@addzeros {00000#1}}%
2902 \def\XINT@dsx@exitvi #1\Z
2903             {\XINT@dsx@addzeros {000000#1}}%
2904 \def\XINT@dsx@exitvii #1\Z
2905             {\XINT@dsx@addzeros {0000000#1}}%
2906 \def\XINT@dsx@addzeros #1#2{ #2#1}%
2907 \def\XINT@dsx@xisPos #1\Z #2%
2908 {%
2909     \XINT@dsx@checksignA #2\Z {#1}}%
2910 }%
2911 \def\XINT@dsx@checksignA #1%
2912 {%
2913     \xint@UDzerominusfork
2914         #1-\dummy \XINT@dsx@AisZero
2915         0#1\dummy \XINT@dsx@AisNeg
2916         0-\dummy {\XINT@dsx@AisPos #1}}%
2917     \xint@UDkrof
2918 }%
2919 \def\XINT@dsx@AisZero #1\Z #2{ {0}{0}}%
2920 \def\XINT@dsx@AisNeg #1\Z #2%
2921 {%
2922     \expandafter
2923         \XINT@dsx@AisNeg@dosplit@andcheckfirst
2924         \romannumerical0\XINT@split@checksize {#2}{#1}}%
2925 }%

```

```

2926 \def\XINT@dsx@AisNeg@dospplit@andcheckfirst #1%
2927 {%
2928     \XINT@dsx@AisNeg@checkiffirstempty #1\Z
2929 }%
2930 \def\XINT@dsx@AisNeg@checkiffirstempty #1%
2931 {%
2932     \xint@z #1\XINT@dsx@AisNeg@finish@zero\Z
2933     \XINT@dsx@AisNeg@finish@notzero #1%
2934 }%
2935 \def\XINT@dsx@AisNeg@finish@zero\Z
2936     \XINT@dsx@AisNeg@finish@notzero\Z #1%
2937 {%
2938     \expandafter
2939         \XINT@dsx@end
2940     \expandafter {\romannumeral0\XINT@num {-#1}}{0}%
2941 }%
2942 \def\XINT@dsx@AisNeg@finish@notzero #1\Z #2%
2943 {%
2944     \expandafter
2945         \XINT@dsx@end
2946     \expandafter {\romannumeral0\XINT@num {#2}}{-#1}%
2947 }%
2948 \def\XINT@dsx@AisPos #1\Z #2%
2949 {%
2950     \expandafter
2951         \XINT@dsx@AisPos@finish
2952     \romannumeral0\XINT@split@checksizex {#2}{#1}%
2953 }%
2954 \def\XINT@dsx@AisPos@finish #1#2%
2955 {%
2956     \expandafter
2957         \XINT@dsx@end
2958     \expandafter {\romannumeral0\XINT@num {#2}}%
2959             {\romannumeral0\XINT@num {#1}}%
2960 }%
2961 \def\XINT@dsx@end #1#2%
2962 {%
2963     \expandafter\space\expandafter{#2}{#1}%
2964 }%

```

12.33 **\xintDecSplit**, **\xintDecSplitL**, **\xintDecSplitR**

DECIMAL SPLIT

v1.01: **New** behavior, for use in future extensions of the **xint** bundle:
The macro **\xintDecSplit {x}{A}** first replaces A with |A| (*)
This macro cuts the number into two pieces L and R. The concatenation LR
always reproduces |A|, and R may be empty or have leading zeros. The
position of the cut is specified by the first argument x. If x is zero or
positive the cut location is x slots to the left of the right end of the
number. If x becomes equal to or larger than the length of the number then L
becomes empty. If x is negative the location of the cut is x slots to the
right of the left end of the number.

(*) warning: this may change in a future version. Only the behavior for A non-negative is guaranteed to remain the same.

```

2965 \def\xintDecSplitL {\romannumeral0\xintdecsplitl }%
2966 \def\xintDecSplitR {\romannumeral0\xintdecsplitr }%
2967 \def\xintdecsplitl
2968 {%
2969     \expandafter
2970         \xint@firstoftwo@andstop
2971     \romannumeral0\xintdecsplit
2972 }%
2973 \def\xintdecsplitr
2974 {%
2975     \expandafter
2976         \xint@secondoftwo@andstop
2977     \romannumeral0\xintdecsplit
2978 }%
2979 \def\xintDecSplit {\romannumeral0\xintdecsplit }%
2980 \def\xintdecsplit #1#2%
2981 {%
2982     \expandafter
2983         \xint@split
2984     \expandafter
2985     {\romannumeral0\xintabs {#2}}{#1}%
2986     fait expansion de A
2986 }%
2987 \def\xint@split #1#2%
2988 {%
2989     \expandafter\expandafter\expandafter
2990         \XINT@split@checksize
2991     \expandafter\expandafter\expandafter
2992     {#2}{#1}%
2993 }%
2994 \def\XINT@split@checksize #1%
2995 {%
2996     \ifnum\XINT@Len {#1} > 9
2997         \xint@afterfi {\xintError:TooBigDecimalSplit
2998                         \XINT@split@bigx }%
2999     \else
3000         \expandafter\XINT@split@xfork
3001     \fi
3002     #1\Z
3003 }%
3004 \def\XINT@split@bigx #1\Z #2%
3005 {%
3006     \ifcase\XINT@Sgn {#1}
3007     \or \xint@afterfi { {}{#2}}%
3008     \else
3009         \xint@afterfi { {#2}{}}%
3010     \fi
3011 }%
3012 \def\XINT@split@xfork #1%
3013 {%
3014     \xint@UDzerominusfork

```

```

3015      #1-\dummy  \XINT@split@zerosplit
3016      0#1\dummy  \XINT@split@fromleft
3017      0-\dummy  {\XINT@split@fromright #1}%
3018      \xint@UDkrof
3019 }%
3020 \def\xint@split@zerosplit #1\Z #2{ {#2}{}}%
3021 \def\xint@split@fromleft #1\Z #2%
3022 {%
3023     \XINT@split@fromleft@loop {#1}{#2}\W\W\W\W\W\W\W\W\W\W\Z
3024 }%
3025 \def\xint@split@fromleft@loop #1%
3026 {%
3027     \ifcase #1
3028         \expandafter\xint@split@fromleft@endsplit
3029     \or
3030         \expandafter\xint@split@fromleft@one@andend
3031     \or
3032         \expandafter\xint@split@fromleft@two@andend
3033     \or
3034         \expandafter\xint@split@fromleft@three@andend
3035     \or
3036         \expandafter\xint@split@fromleft@four@andend
3037     \or
3038         \expandafter\xint@split@fromleft@five@andend
3039     \or
3040         \expandafter\xint@split@fromleft@six@andend
3041     \or
3042         \expandafter\xint@split@fromleft@seven@andend
3043     \else
3044         \expandafter \xint@split@fromleft@loop@perhaps
3045         \expandafter
3046             {\the\numexpr
3047                 #1-8\expandafter\expandafter\expandafter }%
3048         \expandafter
3049         \xint@split@fromleft@eight
3050     \fi
3051 }%
3052 \def\xint@split@fromleft@endsplit #1#2\W #3\Z
3053 { {#1}{#2}}%
3054 \def\xint@split@fromleft@eight #1#2#3#4#5#6#7#8#9%
3055 {%
3056     #9{#1#2#3#4#5#6#7#8#9}%
3057 }%
3058 \def\xint@split@fromleft@loop@perhaps #1#2%
3059 {%
3060     \xint@w #2\xint@split@fromleft@toofar\W \xint@split@fromleft@loop
3061     {#1}%
3062 }%
3063 \def\xint@split@fromleft@toofar\W \xint@split@fromleft@loop #1#2#3\Z
3064 {%
3065     \xint@split@fromleft@toofar@b #2\Z
3066 }%

```

```

3067 \def\xint@split@fromleft@toofar@b #1\W #2\Z
3068 {%
3069   \space {\#1}{}}%
3070 }%
3071 \def\xint@split@fromleft@one@andend
3072   {\expandafter\xint@split@fromleft@checkiftoofar\xint@split@fromleft@one }%
3073 \def\xint@split@fromleft@one #1#2{#2{\#1#2}}%
3074 \def\xint@split@fromleft@two@andend
3075   {\expandafter\xint@split@fromleft@checkiftoofar\xint@split@fromleft@two }%
3076 \def\xint@split@fromleft@two #1#2#3{#3{\#1#2#3}}%
3077 \def\xint@split@fromleft@three@andend
3078   {\expandafter\xint@split@fromleft@checkiftoofar\xint@split@fromleft@three }%
3079 \def\xint@split@fromleft@three #1#2#3#4{#4{\#1#2#3#4}}%
3080 \def\xint@split@fromleft@four@andend
3081   {\expandafter\xint@split@fromleft@checkiftoofar\xint@split@fromleft@four }%
3082 \def\xint@split@fromleft@four #1#2#3#4#5{#5{\#1#2#3#4#5}}%
3083 \def\xint@split@fromleft@five@andend
3084   {\expandafter\xint@split@fromleft@checkiftoofar\xint@split@fromleft@five }%
3085 \def\xint@split@fromleft@five #1#2#3#4#5#6{#6{\#1#2#3#4#5#6}}%
3086 \def\xint@split@fromleft@six@andend
3087   {\expandafter\xint@split@fromleft@checkiftoofar\xint@split@fromleft@six }%
3088 \def\xint@split@fromleft@six #1#2#3#4#5#6#7{#7{\#1#2#3#4#5#6#7}}%
3089 \def\xint@split@fromleft@seven@andend
3090   {\expandafter\xint@split@fromleft@checkiftoofar\xint@split@fromleft@seven }%
3091 \def\xint@split@fromleft@seven #1#2#3#4#5#6#7#8{#8{\#1#2#3#4#5#6#7#8}}%
3092 \def\xint@split@fromleft@checkiftoofar #1#2#3\W #4\Z
3093 {%
3094   \xint@w #1\xint@split@fromleft@wenttoofar\W
3095   \space {\#2}{\#3}}%
3096 }%
3097 \def\xint@split@fromleft@wenttoofar\W\space #1\Z
3098 {%
3099   \xint@split@fromleft@wenttoofar@b #1\Z
3100 }%
3101 \def\xint@split@fromleft@wenttoofar@b #1\W #2\Z
3102 {%
3103   \space {\#1}}%
3104 }%
3105 \def\xint@split@fromright #1\Z #2%
3106 {%
3107   \expandafter
3108     \xint@split@fromright@a
3109   \expandafter
3110   {\romannumeral0\xint@rev {\#2}}{\#1}{\#2}}%
3111 }%
3112 \def\xint@split@fromright@a #1#2%
3113 {%
3114   \xint@split@fromright@loop {\#2}{}#1\W\W\W\W\W\W\W\W\Z
3115 }%
3116 \def\xint@split@fromright@loop #1%
3117 {%
3118   \ifcase #1

```

```

3119      \expandafter\XINT@split@fromright@endsplit
3120      \or
3121      \expandafter\XINT@split@fromright@one@andend
3122      \or
3123      \expandafter\XINT@split@fromright@two@andend
3124      \or
3125      \expandafter\XINT@split@fromright@three@andend
3126      \or
3127      \expandafter\XINT@split@fromright@four@andend
3128      \or
3129      \expandafter\XINT@split@fromright@five@andend
3130      \or
3131      \expandafter\XINT@split@fromright@six@andend
3132      \or
3133      \expandafter\XINT@split@fromright@seven@andend
3134      \else
3135      \expandafter \XINT@split@fromright@loop@perhaps
3136      \expandafter
3137      {\the\numexpr
3138      #1-8\expandafter\expandafter\expandafter }%
3139      \expandafter
3140      \XINT@split@fromright@eight
3141      \fi
3142 }%
3143 \def\XINT@split@fromright@endsplit #1#2\W #3\Z #4%
3144 {%
3145     \expandafter
3146     \space
3147     \expandafter {\romannumeral0\XINT@rev{#2}}{#1}%
3148 }%
3149 \def\XINT@split@fromright@eight #1#2#3#4#5#6#7#8#9%
3150 {%
3151     #9{#9#8#7#6#5#4#3#2#1}%
3152 }%
3153 \def\XINT@split@fromright@loop@perhaps #1#2%
3154 {%
3155     \xint@w #2\XINT@split@fromright@toofar\W\XINT@split@fromright@loop
3156     {#1}%
3157 }%
3158 \def\XINT@split@fromright@toofar\W\XINT@split@fromright@loop #1#2#3\Z
3159     { {} }%
3160 \def\XINT@split@fromright@one@andend
3161     {\expandafter\XINT@split@fromright@checkifsofar\XINT@split@fromright@one }%
3162 \def\XINT@split@fromright@one #1#2{#2{#2#1}}%
3163 \def\XINT@split@fromright@two@andend
3164     {\expandafter\XINT@split@fromright@checkifsofar\XINT@split@fromright@two }%
3165 \def\XINT@split@fromright@two #1#2#3{#3{#3#2#1}}%
3166 \def\XINT@split@fromright@three@andend
3167     {\expandafter\XINT@split@fromright@checkifsofar\XINT@split@fromright@three }%
3168 \def\XINT@split@fromright@three #1#2#3#4{#4{#4#3#2#1}}%
3169 \def\XINT@split@fromright@four@andend
3170     {\expandafter\XINT@split@fromright@checkifsofar\XINT@split@fromright@four }%

```

```

3171 \def\xint@split@fromright@four #1#2#3#4#5{#5{#5#4#3#2#1}}%
3172 \def\xint@split@fromright@five@andend
3173 {\expandafter\xint@split@fromright@checkiftofar\xint@split@fromright@five }%
3174 \def\xint@split@fromright@five #1#2#3#4#5#6{#6{#6#5#4#3#2#1}}%
3175 \def\xint@split@fromright@six@andend
3176 {\expandafter\xint@split@fromright@checkiftofar\xint@split@fromright@six }%
3177 \def\xint@split@fromright@six #1#2#3#4#5#6#7{#7{#7#6#5#4#3#2#1}}%
3178 \def\xint@split@fromright@seven@andend
3179 {\expandafter\xint@split@fromright@checkiftofar\xint@split@fromright@seven }%
3180 \def\xint@split@fromright@seven #1#2#3#4#5#6#7#8{#8{#8#7#6#5#4#3#2#1}}%
3181 \def\xint@split@fromright@checkiftofar #1%
3182 {%
3183     \xint@w #1\xint@split@fromright@wenttofar\W
3184     \xint@split@fromright@endsplit
3185 }%
3186 \def\xint@split@fromright@wenttofar\W\xint@split@fromright@endsplit #1\Z #2%
3187 { {}{#2}}%
3188 \xint@restorecatcodes@endinput%

```

13 Package **xintgcd** implementation

The commenting is currently (2013/04/04) very sparse.

13.1 Catcodes, ε -**T_EX** detection, reload detection

The code for reload detection is copied from HEIKO OBERDIEK's packages, and adapted here to check for previous loading of the master **xint** package.

The method for catcodes is slightly different, but still directly inspired by these packages.

```

3189 \begingroup\catcode61\catcode48\catcode32=10\relax%
3190 \catcode13=5 % ^M
3191 \endlinechar=13 %
3192 \catcode123=1 % {
3193 \catcode125=2 % }
3194 \catcode64=11 % @
3195 \catcode35=6 % #
3196 \catcode44=12 % ,
3197 \catcode45=12 % -
3198 \catcode46=12 % .
3199 \catcode58=12 % :
3200 \def\space { }%
3201 \let\z\endgroup
3202 \expandafter\let\expandafter\x\csname ver@xintgcd.sty\endcsname
3203 \expandafter\let\expandafter\w\csname ver@xint.sty\endcsname
3204 \expandafter
3205     \ifx\csname PackageInfo\endcsname\relax
3206         \def\y#1#2{\immediate\write-1{Package #1 Info: #2.}}%
3207     \else
3208         \def\y#1#2{\PackageInfo{#1}{#2}}%
3209     \fi
3210 \expandafter
3211 \ifx\csname numexpr\endcsname\relax

```

```

3212   \y{xintgcd}{\numexpr not available, aborting input}%
3213   \aftergroup\endinput
3214 \else
3215   \ifx\x\relax % plain-TeX, first loading of xintgcd.sty
3216     \ifx\w\relax % but xint.sty not yet loaded.
3217       \y{xintgcd}{Package xint is required}%
3218       \y{xintgcd}{Will try \string\input\space xint.sty}%
3219       \def\z{\endgroup\input xint.sty\relax}%
3220     \fi
3221   \else
3222     \def\empty {}%
3223     \ifx\x\empty % LaTeX, first loading,
3224       % variable is initialized, but \ProvidesPackage not yet seen
3225       \ifx\w\relax % xint.sty not yet loaded.
3226         \y{xintgcd}{Package xint is required}%
3227         \y{xintgcd}{Will try \string\RequirePackage{xint}}%
3228         \def\z{\endgroup\RequirePackage{xint}}%
3229       \fi
3230     \else
3231       \y{xintgcd}{I was already loaded, aborting input}%
3232       \aftergroup\endinput
3233     \fi
3234   \fi
3235 \fi
3236 \z%

```

13.2 Validation of **xint** loading

```

3237 \begingroup\catcode61\catcode48\catcode32=10\relax%
3238   \catcode13=5    % ^^M
3239   \endlinechar=13 %
3240   \catcode123=1   % {
3241   \catcode125=2   % }
3242   \catcode64=11   % @
3243   \catcode35=6    % #
3244   \catcode44=12   % ,
3245   \catcode45=12   % -
3246   \catcode46=12   % .
3247   \catcode58=12   % :
3248 \expandafter
3249   \ifx\csname PackageInfo\endcsname\relax
3250     \def\y#1#2{\immediate\write-1{Package #1 Info: #2.}}%
3251   \else
3252     \def\y#1#2{\PackageInfo{#1}{#2}}%
3253   \fi
3254 \def\empty {}%
3255 \expandafter\let\expandafter\w\csname ver@xint.sty\endcsname
3256 \ifx\w\relax % Plain TeX, user gave a file name at the prompt
3257   \y{xintgcd}{Loading of package xint failed, aborting input}%
3258   \aftergroup\endinput
3259 \fi
3260 \ifx\w\empty % LaTeX, user gave a file name at the prompt

```

```

3261      \y{xintgcd}{Loading of package xint failed, aborting input}%
3262      \aftergroup\endinput
3263  \fi
3264 \endgroup%

```

13.3 Catcodes

Perhaps catcodes have changed after the loading of **xint** and prior to the current loading of **xintgcd**, so we can not employ the `\XINT@restorecatcodes@endinput` in this style file. But there is no problem using `\XINT@setcatcodes`.

```

3265 \begingroup\catcode61\catcode48\catcode32=10\relax%
3266  \catcode13=5    % ^M
3267  \endlinechar=13 %
3268  \catcode123=1   % {
3269  \catcode125=2   % }
3270  \catcode64=11   % @
3271  \def\x
3272  {%
3273      \endgroup
3274      \edef\XINT@gcd@restorecatcodes@endinput
3275      {%
3276          \catcode36=\the\catcode36    % $
3277          \catcode47=\the\catcode47    % /
3278          \catcode41=\the\catcode41    % )
3279          \catcode40=\the\catcode40    % (
3280          \catcode42=\the\catcode42    % *
3281          \catcode43=\the\catcode43    % +
3282          \catcode62=\the\catcode62    % >
3283          \catcode60=\the\catcode60    % <
3284          \catcode58=\the\catcode58    % :
3285          \catcode46=\the\catcode46    % .
3286          \catcode45=\the\catcode45    % -
3287          \catcode44=\the\catcode44    % ,
3288          \catcode35=\the\catcode35    % #
3289          \catcode64=\the\catcode64    % @
3290          \catcode125=\the\catcode125 % }
3291          \catcode123=\the\catcode123 % {
3292          \endlinechar=\the\endlinechar
3293          \catcode13=\the\catcode13    % ^M
3294          \catcode32=\the\catcode32    %
3295          \catcode61=\the\catcode61    % =
3296          \noexpand\endinput
3297      }%
3298      \XINT@setcatcodes
3299      \catcode36=3    % $
3300  }%
3301 \x

```

13.4 Package identification

```

3302 \begingroup
3303  \catcode91=12 % [

```

```

3304  \catcode93=12 % ]
3305  \catcode58=12 % :
3306  \expandafter\ifx\csname ProvidesPackage\endcsname\relax
3307    \def\x#1#2#3[#4]{\endgroup
3308      \immediate\write-1{Package: #3 #4}%
3309      \xdef#1[#4]%
3310    }%
3311  \else
3312    \def\x#1#2[#3]{\endgroup
3313      #2[{#3}]%
3314      \ifx#1\undefined
3315        \xdef#1[#3]%
3316      \fi
3317      \ifx#1\relax
3318        \xdef#1[#3]%
3319      \fi
3320    }%
3321  \fi
3322 \expandafter\x\csname ver@xintgcd.sty\endcsname
3323 \ProvidesPackage{xintgcd}%
3324 [2013/04/04 v1.01 Euclide algorithm with xint package (jfb)]%

```

13.5 \xintGCD

```

3325 \def\xintGCD {\romannumeral0\xintgcd }%
3326 \def\xintgcd #1%
3327 {%
3328   \expandafter
3329   \XINT@gcd
3330   \expandafter
3331   {\romannumeral0\xintabs {#1}}%
3332 }%
3333 \def\XINT@gcd #1#2%
3334 {%
3335   \expandafter
3336   \XINT@gcd@fork
3337   \romannumeral0\xintabs {#2}\Z #1\Z
3338 }%
Ici #3#4=A, #1#2=B
3339 \def\XINT@gcd@fork #1#2\Z #3#4\Z
3340 {%
3341   \xint@UDzerofork
3342   #1\dummy \XINT@gcd@BisZero
3343   #3\dummy \XINT@gcd@AisZero
3344   0\dummy \XINT@gcd@loop
3345   \xint@UDkrof
3346   {#1#2}{#3#4}%
3347 }%
3348 \def\XINT@gcd@AisZero #1#2{ #1}%
3349 \def\XINT@gcd@BisZero #1#2{ #2}%
3350 \def\XINT@gcd@CheckRem #1#2\Z
3351 {%
3352   \xint@zero #1\xint@gcd@end0\XINT@gcd@loop {#1#2}%

```

```

3353 }%
3354 \def\xint@gcd@end@XINT@gcd@loop #1#2{ #2}%
#1=B, #2=A
3355 \def\XINT@gcd@loop #1#2%
3356 {%
3357     \expandafter\expandafter\expandafter
3358         \XINT@gcd@CheckRem
3359     \expandafter\xint@secondoftwo
3360     \romannumeral0\XINT@div@prepare {#1}{#2}\Z
3361     {#1}%
3362 }%

```

13.6 \xintBezout

```

3363 \def\xintBezout {\romannumeral0\xintbezout }%
3364 \def\xintbezout #1%
3365 {%
3366     \expandafter\expandafter\expandafter
3367         \xint@bezout
3368     \expandafter\expandafter\expandafter
3369     {#1}%
3370 }%
3371 \def\xint@bezout #1#2%
3372 {\expandafter\expandafter\expandafter
3373     \XINT@bezout@fork #2\Z #1\Z
3374 }%
#3#4 = A, #1#2=B
3375 \def\XINT@bezout@fork #1#2\Z #3#4\Z
3376 {%
3377     \xint@UDzerosfork
3378     #1#3\dummy \XINT@bezout@botharezero
3379     #10\dummy \XINT@bezout@secondiszero
3380     #30\dummy \XINT@bezout@firstiszero
3381     00\dummy
3382     {\xint@UDsignsfork
3383         #1#3\dummy \XINT@bezout@minusminus % A < 0, B < 0
3384         #1-\dummy \XINT@bezout@minusplus % A > 0, B < 0
3385         #3-\dummy \XINT@bezout@plusminus % A < 0, B > 0
3386         --\dummy \XINT@bezout@plusplus % A > 0, B > 0
3387     \xint@UDkrof }%
3388     \xint@UDkrof
3389     {#2}{#4}#1#3{#3#4}{#1#2}% #1#2=B, #3#4=A
3390 }%
3391 \def\XINT@bezout@botharezero #1#2#3#4#5#6%
3392 {%
3393     \xintError:NoBezoutForZeros
3394     \space {0}{0}{0}{0}{0}{0}%
3395 }%
attention première entrée doit être ici  $(-1)^n$  donc 1
#4#2=0 = A, B = #3#1

```

13 Package *xintgcd* implementation

```

3396 \def\XINT@bezout@firstiszero #1#2#3#4#5#6%
3397 {%
3398     \xint@UDsignfork
3399     #3\dummy { {0}{#3#1}{0}{1}{#1} }%
3400     -\dummy { {0}{#3#1}{0}{-1}{#1} }%
3401     \xint@UDkrof
3402 }%
3403 %
#4#2= A, B = #3#1 = 0

3403 \def\XINT@bezout@secondiszero #1#2#3#4#5#6%
3404 {%
3405     \xint@UDsignfork
3406     #4\dummy{ {#4#2}{0}{-1}{0}{#2} }%
3407     -\dummy{ {#4#2}{0}{1}{0}{#2} }%
3408     \xint@UDkrof
3409 }%
3410 %
#4#2= A < 0, #3#1 = B < 0

3410 \def\XINT@bezout@minusminus #1#2#3#4%
3411 {%
3412     \expandafter\XINT@bezout@mm@post
3413     \romannumeral0\XINT@bezout@loop@a 1{#1}{#2}1001%
3414 }%
3415 \def\XINT@bezout@mm@post #1#2%
3416 {%
3417     \expandafter
3418     \XINT@bezout@mm@postb
3419     \expandafter
3420     {\romannumeral0\xintopp{#2}}{\romannumeral0\xintopp{#1}}%
3421 }%
3422 \def\XINT@bezout@mm@postb #1#2%
3423 {%
3424     \expandafter
3425     \XINT@bezout@mm@postc
3426     \expandafter {#2}{#1}%
3427 }%
3428 \def\XINT@bezout@mm@postc #1#2#3#4#5%
3429 {%
3430     \space {#4}{#5}{#1}{#2}{#3}%
3431 }%
3432 %
minusplus #4#2= A > 0, B < 0

3432 \def\XINT@bezout@minusplus #1#2#3#4%
3433 {%
3434     \expandafter\XINT@bezout@mp@post
3435     \romannumeral0\XINT@bezout@loop@a 1{#1}{#4#2}1001%
3436 }%
3437 \def\XINT@bezout@mp@post #1#2%
3438 {%
3439     \expandafter
3440     \XINT@bezout@mp@postb

```

13 Package **xintgcd** implementation

```

3441      \expandafter
3442          {\romannumeral0\xintopp {\#2}}{\#1}%
3443 }%
3444 \def\xint@bezout@mp@postb #1#2#3#4#5%
3445 {%
3446     \space {\#4}{\#5}{\#2}{\#1}{\#3}%
3447 }%

plusminus A < 0, B > 0

3448 \def\xint@bezout@plusminus #1#2#3#4%
3449 {%
3450     \expandafter\xint@bezout@pm@post
3451     \romannumeral0\xint@bezout@loop@a 1{\#3#1}{\#2}1001%
3452 }%
3453 \def\xint@bezout@pm@post #1%
3454 {%
3455     \expandafter
3456         \xint@bezout@pm@postb
3457     \expandafter
3458         {\romannumeral0\xintopp{\#1}}%
3459 }%
3460 \def\xint@bezout@pm@postb #1#2#3#4#5%
3461 {%
3462     \space {\#4}{\#5}{\#1}{\#2}{\#3}%
3463 }%

plusplus

3464 \def\xint@bezout@plusplus #1#2#3#4%
3465 {%
3466     \expandafter\xint@bezout@pp@post
3467     \romannumeral0\xint@bezout@loop@a 1{\#3#1}{\#4#2}1001%
3468 }%

la parité  $(-1)^N$  est en #1, et on la jette ici.

3469 \def\xint@bezout@pp@post #1#2#3#4#5%
3470 {%
3471     \space {\#4}{\#5}{\#1}{\#2}{\#3}%
3472 }%

n = 0: 1BAalpha(0)beta(0)alpha(-1)beta(-1)
n général:
{(-1)^n}{r(n-1)}{r(n-2)}{alpha(n-1)}{beta(n-1)}{alpha(n-2)}{beta(n-2)}
#2 = B, #3 = A

3473 \def\xint@bezout@loop@a #1#2#3%
3474 {%
3475     \expandafter\xint@bezout@loop@b
3476     \expandafter{\the\numexpr -#1\expandafter }%
3477     \romannumeral0\xint@div@prepare {\#2}{\#3}{\#2}%
3478 }%

```

13 Package **xintgcd** implementation

Le $q(n)$ a ici une existence éphémère, dans le version Bezout Algorithm il faudra le conserver. On voudra à la fin
 $\{{\{q(n)\}{r(n)}{\alpha(n)}{\beta(n)}}\}$
 De plus ce n'est plus $(-1)^n$ que l'on veut mais n . (ou dans un autre ordre)
 $\{-(-1)^n\{q(n)\}{r(n)}\{r(n-1)\}{\alpha(n-1)}{\beta(n-1)}\{\alpha(n-2)\}{\beta(n-2)}\}$

```

3479 \def\xint@bezout@loop@b #1#2#3#4#5#6#7#8%
3480 {%
3481     \expandafter
3482         \xint@bezout@loop@c
3483     \expandafter
3484         {\romannumeral0\xintadd{\xint@Mul{#5}{#2}}{#7}}%
3485         {\romannumeral0\xintadd{\xint@Mul{#6}{#2}}{#8}}%
3486     {#1}{#3}{#4}{#5}{#6}%
3487 }%
3488 {\alpha(n)}{->}\beta(n){-(-1)^n}{r(n)}{r(n-1)}{\alpha(n-1)}{\beta(n-1)}
3489 \def\xint@bezout@loop@c #1#2%
3490 {%
3491     \expandafter
3492         \xint@bezout@loop@d
3493     \expandafter
3494     {#2}{#1}%
3495 }%
3496 {\beta(n)}{\alpha(n)}{(-1)^(n+1)}{r(n)}{r(n-1)}{\alpha(n-1)}{\beta(n-1)}
3497 \def\xint@bezout@loop@d #1#2#3#4#5%
3498 {%
3499     \xint@zero #1\xint@bezout@loop@exit0\xint@bezout@loop@f
3500     {#1#2}%
3501 }%
3502 {\r(n)\Z}{(-1)^(n+1)}{r(n-1)}{\alpha(n)}{\beta(n)}{\alpha(n-1)}{\beta(n-1)}
3503 \def\xint@bezout@loop@e #1#2\Z
3504 {%
3505     \xint@zero #1\xint@bezout@loop@exit0\xint@bezout@loop@f
3506     {#1#2}%
3507 }%
3508 {\r(n)}{(-1)^(n+1)}{r(n-1)}{\alpha(n)}{\beta(n)}{\alpha(n-1)}{\beta(n-1)}
3509 \def\xint@bezout@loop@f #1#2%
3510 {%
3511     \xint@zero #1\xint@bezout@loop@a {#2}{#1}%
3512 }%
3513 {(-1)^(n+1)}{r(n)}{r(n-1)}{\alpha(n)}{\beta(n)}{\alpha(n-1)}{\beta(n-1)}
et itération

```

```

3508 \def\xint@bezout@loop@exit0\XINT@bezout@loop@f #1#2%
3509 {%
3510     \ifcase #2
3511         \or \expandafter\XINT@bezout@exiteven
3512         \else\expandafter\XINT@bezout@exitodd
3513     \fi
3514 }%
3515 \def\XINT@bezout@exiteven #1#2#3#4#5%
3516 {%
3517     \space {#5}{#4}{#1}%
3518 }%
3519 \def\XINT@bezout@exitodd #1#2#3#4#5%
3520 {%
3521     \space {-#5}{-#4}{#1}%
3522 }%

```

13.7 \xintEuclideAlgorithm

Pour Euclide:

$\{N\}\{A\}\{D=r(n)\}\{B\}\{q1\}\{r1\}\{q2\}\{r2\}\{q3\}\{r3\}\dots\{qN\}\{rN=0\}$
 $u<2n> = u<2n+3>u<2n+2> + u<2n+4>$ à la n ième étape

```

3523 \def\xintEuclideAlgorithm {\romannumeral0\xinteclidalgorithm }%
3524 \def\xinteclidalgorithm #1%
3525 {%
3526     \expandafter
3527         \XINT@euc
3528     \expandafter
3529         {\romannumeral0\xintabs {#1}}%
3530 }%
3531 \def\XINT@euc #1#2%
3532 {%
3533     \expandafter
3534         \XINT@euc@fork
3535     \romannumeral0\xintabs {#2}\Z #1\Z
3536 }%

```

Ici #3#4=A, #1#2=B

```

3537 \def\XINT@euc@fork #1#2\Z #3#4\Z
3538 {%
3539     \xint@UDzerofork
3540         #1\dummy \XINT@euc@BisZero
3541         #3\dummy \XINT@euc@AisZero
3542         0\dummy \XINT@euc@a
3543     \xint@UDkrof
3544     {0}\{#1#2}\{#3#4}\{#3#4}\{#1#2}\}\}\Z
3545 }%

```

Le {} pour protéger {{A}{B}} si on s'arrête après une étape (B divise A)
On va renvoyer:

$\{N\}\{A\}\{D=r(n)\}\{B\}\{q1\}\{r1\}\{q2\}\{r2\}\{q3\}\{r3\}\dots\{qN\}\{rN=0\}$

13 Package *xintgcd* implementation

```

3546 \def\XINT@euc@AisZero #1#2#3#4#5#6{ {1}{0}{{#2}{#2}{0}{0}} }%
3547 \def\XINT@euc@BisZero #1#2#3#4#5#6{ {1}{0}{{#3}{#3}{0}{0}} }%

{n}{rn}{an}{{qn}{rn}}...{{A}{B}}}\Z
an = r(n-1)
Pour n=0 on a juste {0}{B}{A}{{A}{B}}}\Z
\XINT@div@prepare {u}{v} divise v par u

3548 \def\XINT@euc@a #1#2#3%
3549 {%
3550     \expandafter
3551         \XINT@euc@b
3552     \expandafter {\the\numexpr #1+1\expandafter }%
3553     \romannumeral0\XINT@div@prepare {{#2}{#3}{#2}}%
3554 }%

{n+1}{q(n+1)}{r(n+1)}{rn}{{qn}{rn}}...

3555 \def\XINT@euc@b #1#2#3#4%
3556 {%
3557     \XINT@euc@c #3\Z {{#1}{#3}{#4}{##2}{#3}}%
3558 }%

r(n+1)\Z {n+1}{r(n+1)}{r(n)}{{q(n+1)}{r(n+1)}}{{qn}{rn}}...
Test si r(n+1) est nul.

3559 \def\XINT@euc@c #1#2\Z
3560 {%
3561     \xint@zero #1\xint@euc@end0\XINT@euc@a
3562 }%

{n+1}{r(n+1)}{r(n)}{{q(n+1)}{r(n+1)}}...}\Z
Ici r(n+1) = 0. On arrête on se prépare à inverser.
{n+1}{0}{r(n)}{{q(n+1)}{r(n+1)}}....{{q1}{r1}}{{A}{B}}}\Z
On veut renvoyer:
{N=n+1}{A}{D=r(n)}{B}{q1}{r1}{q2}{r2}{q3}{r3}....{qN}{rN=0}

3563 \def\xint@euc@end0\XINT@euc@a #1#2#3#4\Z%
3564 {%
3565     \expandafter\xint@euc@end@
3566     \romannumeral0%
3567     \XINT@rord@main {{#4}{#1}{#3}}%
3568     \xint@UNDEF
3569     \xint@undef\xint@undef\xint@undef\xint@undef
3570     \xint@undef\xint@undef\xint@undef\xint@undef
3571     \xint@UNDEF
3572 }%
3573 \def\xint@euc@end@ #1#2#3%
3574 {%
3575     \space {{#1}{#3}{#2}}%
3576 }%

```

13.8 \xintBezoutAlgorithm

```

Pour Bezout: objectif, renvoyer
alpha0=1, beta0=0
alpha(-1)=0, beta(-1)=1
{N}{A}{0}{1}{D=r(n)}{B}{1}{0}{q1}{r1}{alpha1=q1}{beta1=1}
{q2}{r2}{alpha2}{beta2}....{qN}{rN=0}{alphaN=A/D}{betaN=B/D}

3577 \def\xintBezoutAlgorithm {\romannumeral0\xintbezoutalgorithm }%
3578 \def\xintbezoutalgorithm #1%
3579 {%
3580     \expandafter
3581         \XINT@bezalg
3582     \expandafter
3583         {\romannumeral0\xintabs {\#1}}%
3584 }%
3585 \def\XINT@bezalg #1#2%
3586 {%
3587     \expandafter
3588         \XINT@bezalg@fork
3589     \romannumeral0\xintabs {\#2}\Z #1\Z
3590 }%

Ici #3#4=A, #1#2=B

3591 \def\XINT@bezalg@fork #1#2\Z #3#4\Z
3592 {%
3593     \xint@UDzerofork
3594     #1\dummy \XINT@bezalg@BisZero
3595     #3\dummy \XINT@bezalg@AisZero
3596     0\dummy \XINT@bezalg@a
3597     \xint@UDkrof
3598     0{\#1#2}{#3#4}1001{\#3#4}{#1#2}{}{}\Z
3599 }%
3600 \def\XINT@bezalg@AisZero #1#2#3\Z{ {1}{0}{0}{1}{#2}{#2}{1}{0}{0}{0}{0}{1}}%
3601 \def\XINT@bezalg@BisZero #1#2#3#4\Z{ {1}{0}{0}{1}{#3}{#3}{1}{0}{0}{0}{1}}%

pour préparer l'étape n+1 il faut
{ n } { r(n) } { r(n-1) } { alpha(n) } { beta(n) } { alpha(n-1) } { beta(n-1) }
{ { q(n) } { r(n) } { alpha(n) } { beta(n) } } ...
division de #3 par #2

3602 \def\XINT@bezalg@a #1#2#3%
3603 {%
3604     \expandafter
3605         \XINT@bezalg@b
3606     \expandafter {\the\numexpr #1+1\expandafter }%
3607     \romannumeral0\XINT@div@prepare {\#2}{#3}{#2}%
3608 }%
{ n+1 } { q(n+1) } { r(n+1) } { r(n) } { alpha(n) } { beta(n) } { alpha(n-1) } { beta(n-1) } ...

```

13 Package *xintgcd* implementation

```

3609 \def\XINT@bezalg@b #1#2#3#4#5#6#7#8%
3610 {%
3611     \expandafter\XINT@bezalg@c\expandafter
3612     {\romannumeral0\xintadd {\xintMul {#6}{#2}}{#8}}%
3613     {\romannumeral0\xintadd {\xintMul {#5}{#2}}{#7}}%
3614     {#1}{#2}{#3}{#4}{#5}{#6}}%
3615 }%
{beta(n+1)}{alpha(n+1)}{n+1}{q(n+1)}{r(n+1)}{r(n)}{alpha(n)}{beta(n)}%
3616 \def\XINT@bezalg@c #1#2#3#4#5#6%
3617 {%
3618     \expandafter\XINT@bezalg@d\expandafter
3619     {#2}{#3}{#4}{#5}{#6}{#1}}%
3620 }%
{alpha(n+1)}{n+1}{q(n+1)}{r(n+1)}{r(n)}{beta(n+1)}%
3621 \def\XINT@bezalg@d #1#2#3#4#5#6#7#8%
3622 {%
3623     \XINT@bezalg@e #4\Z {#2}{#4}{#5}{#1}{#6}{#7}{#8}{##3}{#4}{#1}{#6}}%
3624 }%
r(n+1)\Z {n+1}{r(n+1)}{r(n)}{alpha(n+1)}{beta(n+1)}%
{alpha(n)}{beta(n)}{q,r,alpha,beta(n+1)}%
Test si r(n+1) est nul.
3625 \def\XINT@bezalg@e #1#2\Z
3626 {%
3627     \xint@zero #1\xint@bezalg@end0\XINT@bezalg@a
3628 }%
Ici r(n+1) = 0. On arrête on se prépare à inverser.
{n+1}{r(n+1)}{r(n)}{alpha(n+1)}{beta(n+1)}%
{alpha(n)}{beta(n)}%
{q,r,alpha,beta(n+1)}...{{A}{B}}{}{\Z}
On veut renvoyer
{N}{A}{0}{1}{D=r(n)}{B}{1}{0}{q1}{r1}{alpha1=q1}{beta1=1}%
{q2}{r2}{alpha2}{beta2}...{qN}{rN=0}{alphaN=A/D}{betaN=B/D}
3629 \def\xint@bezalg@end0\XINT@bezalg@a #1#2#3#4#5#6#7#8\Z
3630 {%
3631     \expandafter\xint@bezalg@end@
3632     \romannumeral0%
3633     \XINT@rord@main {}#8{##1}{#3}}%
3634     \xint@UNDEF
3635     \xint@undef\xint@undef\xint@undef\xint@undef
3636     \xint@undef\xint@undef\xint@undef\xint@undef
3637     \xint@UNDEF
3638 }%
{N}{D}{A}{B}{q1}{r1}{alpha1=q1}{beta1=1}{q2}{r2}{alpha2}{beta2}%
...{qN}{rN=0}{alphaN=A/D}{betaN=B/D}
On veut renvoyer
{N}{A}{0}{1}{D=r(n)}{B}{1}{0}{q1}{r1}{alpha1=q1}{beta1=1}%
{q2}{r2}{alpha2}{beta2}...{qN}{rN=0}{alphaN=A/D}{betaN=B/D}
3639 \def\xint@bezalg@end@ #1#2#3#4%
3640 {%
3641     \space {#1}{#3}{0}{1}{#2}{#4}{1}{0}}%
3642 }%

```

13.9 \xintTypesetEuclideAlgorithm

```

TYPESETTING
Organisation:
{N}{A}{D}{B}{q1}{r1}{q2}{r2}{q3}{r3}...{qN}{rN=0}
\U1 = N = nombre d'étapes, \U3 = PGCD, \U2 = A, \U4=B
q1 = \U5, q2 = \U7 --> qn = \U<2n+3>, rn = \U<2n+4>
bn = rn. B = r0. A=r(-1)
r(n-2) = q(n)r(n-1)+r(n) (n e étape) (n au moins 1)
\U{2n} = \U{2n+3} \times \U{2n+2} + \U{2n+4}, n e étape.
avec n entre 1 et N.

3643 \def\xintTypesetEuclideAlgorithm #1#2%
3644 {% l'algo remplace #1 et #2 par |#1| et |#2|
3645   \par
3646   \begingroup
3647     \xintAssignArray\xintEuclideAlgorithm {#1}{#2}\to\U
3648     \edef\A{\U2}\edef\B{\U4}\edef\N{\U1}%
3649     \setbox0\vbox{\halign {$##$\cr \A\cr \B \cr}}%
3650     \noindent
3651     \count 255 1
3652     \loop
3653       \hbox to \wd0 {\hfil\U{\the\numexpr 2*\count 255\relax} $}%
3654       ${} = \U{\the\numexpr 2*\count 255 + 3\relax}
3655       \times \U{\the\numexpr 2*\count 255 + 2\relax}
3656       + \U{\the\numexpr 2*\count 255 + 4\relax} $%
3657     \ifnum \count 255 < \N
3658       \hfill\break
3659       \advance \count 255 1
3660     \repeat
3661   \par
3662   \endgroup
3663 }%

```

13.10 \xintTypesetBezoutAlgorithm

Pour Bezout on a:

```

{N}{A}{0}{1}{D=r(n)}{B}{1}{0}{q1}{r1}{alpha1=q1}{beta1=1}
{q2}{r2}{alpha2}{beta2}...{qN}{rN=0}{alphaN=A/D}{betaN=B/D}%
Donc 4N+8 termes
U1 = N, U2= A, U5=D, U6=B,
q1 = U9, qn = U{4n+5}, n au moins 1
rn = U{4n+6} , n au moins -1
alpha(n) = U{4n+7}, n au moins -1
beta(n) = U{4n+8}, n au moins -1

```

```

3664 \def\xintTypesetBezoutAlgorithm #1#2%
3665 {%
3666   \par
3667   \begingroup
3668     \parindent0pt
3669     \xintAssignArray\xintBezoutAlgorithm {#1}{#2}\to\BEZ
3670     \edef\A{\BEZ2}\edef\B{\BEZ6}\edef\N{\BEZ1}% A = |#1|, B = |#2|

```

```

3671 \setbox 0 \vbox{\halign {$##\cr \A\cr \B \cr} }%
3672 \count 255 1
3673 \loop
3674   \noindent
3675   \hbox to \wd 0 {\hfil$ \BEZ{\the\numexpr 4*\count 255 - 2\relax} $}%
3676   ${} = \BEZ{\the\numexpr 4*\count 255 + 5\relax}
3677   \times \BEZ{\the\numexpr 4*\count 255 + 2\relax}
3678     + \BEZ{\the\numexpr 4*\count 255 + 6\relax} $\hfill\break
3679   \hbox to \wd 0 {\hfil$ \BEZ{\the\numexpr 4*\count 255 + 7\relax} $}%
3680   ${} = \BEZ{\the\numexpr 4*\count 255 + 5\relax}
3681   \times \BEZ{\the\numexpr 4*\count 255 + 3\relax}
3682     + \BEZ{\the\numexpr 4*\count 255 - 1\relax} $\hfill\break
3683   \hbox to \wd 0 {\hfil$ \BEZ{\the\numexpr 4*\count 255 + 8\relax} $}%
3684   ${} = \BEZ{\the\numexpr 4*\count 255 + 5\relax}
3685   \times \BEZ{\the\numexpr 4*\count 255 + 4\relax}
3686     + \BEZ{\the\numexpr 4*\count 255 \relax} $
3687 \endgraf
3688 \ifnum \count 255 < \N
3689   \advance \count 255 1
3690 \repeat
3691 \par
3692 \edef\U{\BEZ{\the\numexpr 4*\N + 4\relax}}%
3693 \edef\V{\BEZ{\the\numexpr 4*\N + 3\relax}}%
3694 \edef\D{\BEZ{5}}%
3695 \ifodd\N\relax
3696   $ \U \times \A - \V \times \B = - \D %
3697 \else
3698   $ \U \times \A - \V \times \B = \D %
3699 \fi
3700 \par
3701 \endgroup
3702 }%
3703 \XINT@gcd@restorecatcodes@endinput%

```